

How do Data Scientists use mathematics?



Aditya Pandey
M.Sc IV Sem

A Data Scientist's primary role is to mine, examine, and make sense of data. Maths plays a role in each of these stages.

Data Scientists use mathematical skills to:

Understand and use machine learning algorithms
Analyse datasets from various sources
Identify patterns in data
Forecast trends and growth
Data Scientists also use mathematical functions to perform data analysis and apply machine learning techniques like clustering, regression, and classification.

Clustering

Clustering is a way to organise data into clusters or groups that share similarities with each other. It involves some calculus and statistics.

A clustering algorithm organises data into these groups to identify trends and reveal insights at the surface level.

For example, a company with a large customer base can use clustering to segment customers based on their demographics or areas of interest. When you are promoting products, you can better personalise your marketing

messages based on data points like customer location, behaviour, interests, and more.

Regression

Regression analysis is a way to measure how certain factors impact outcomes or objectives. In other words, it shows how one variable impacts another. It uses a combination of algebra and statistics.

Data Scientists use regression to make data-driven predictions and help businesses make better decisions. For example, they can use regression to forecast future sales or to predict if a company should increase the inventory of a product.

Classification

Data classification is the process of labelling or categorising data to easily store, retrieve, and use it to predict future outcomes. In machine learning, classification uses a set of training data to organise data into classes. For instance, an email spam filter uses classification to detect if an email is spam or not.

Foundations of data analysis

All data professionals need a solid grasp of essential mathematical concepts, but that's only part of the skill set needed to

analyse data effectively. The ability to work with diverse types of information and create data visualisations are also crucial for gaining meaningful insights.

Working with different data types

Data Analysts and Data Scientists handle a wide range of data types, including:

Categorical data: Qualitative information that can be represented by a name or symbol, such as customer demographics and types of products

Numerical data: Quantitative information, such as conversion rates and sales revenue

You should know how to use Structured Query Language (SQL) to manage categorical and numerical data. This language allows you to query, organise, and filter information in relational databases.

Data visualisation

Data Scientists often transform datasets into accessible graphic representations.

These visualizations can reveal previously unnoticed patterns or anomalies in datasets. They also allow data professionals to communicate their findings with non-technical stakeholders.

Platforms like Microsoft BI and Tableau use machine learning models and mathematics to analyse data. They also have intuitive interfaces that allow you to design interactive dashboards and data visualisations. For example, you

could use line graphs to represent economic trends over time.

You should also learn how to use data visualisation libraries in Python. Popular frameworks include Glean, Matplotlib, and Plotly. They have built-in templates and themes that you can use to create polished visualisations quickly.

What types of maths do Data Scientists need to know?

Luckily, you don't need to be a mathematician or have a Ph.D. in mathematics to be a Data Scientist. Data Scientists use three main types of maths—linear algebra, calculus, and statistics. Probability is another maths data scientists use, but it is sometimes grouped together with statistics.

Linear algebra

Some consider Linear Algebra the mathematics of data and the foundation of machine learning. Data Scientists manipulate and analyse raw data through matrices, rows, and columns of numbers or data points.

Datasets usually take the form of matrices. Data Scientists store and manipulate data inside them and they use linear algebra during the process. For example, linear algebra is a core component of data preprocessing. It's the process of organising raw data so that it can be read and understood by machines.

At a minimum, Data Scientists should know matrices and vectors and how to apply basic algebra principles to solve data problems.

Calculus

Data Scientists use calculus to analyse rates of change and relationships within datasets. These maths skills help them understand how a change in one variable — such as changing customer preferences — affects another variable, like sales revenue.

Before you begin your data science journey, you should master the two main branches of calculus: differential and integral.

- **Differential calculus**

Differential calculus studies how quickly quantities change. Data Scientists should learn its foundational concepts, including limits and derivatives. Python libraries like NumPy and SymPy can speed up this learning process by performing complex calculations efficiently.

Data professionals apply differential calculus to optimise machine learning models and functions. For instance, gradient descent calculates the error between the predicted and actual results. This method allows neural networks and other types of algorithms to adjust their parameters iteratively, reducing errors and improving performance.

- **Integral calculus**

Integral calculus analyses the accumulation of quantities over a

specific integral. To effectively apply this technique, you must understand definite and indefinite integrals. Familiarity with Python libraries like SciPy can also help you calculate integrals.

Data professionals use this branch of mathematics to solve many problems in data science, such as forecasting the demand for a product and analysing revenue. Machine learning algorithms also use integral calculus to calculate probability and variance.

Probability and statistics

Probability and statistics go hand in hand. Data professionals use these mathematical foundations to analyse information and forecast events.

Statistics is the branch of mathematics that collects and analyses large data sets to extract meaningful insights from them. Data Scientists use statistics to:

- Collect, review, analyse, and form insights from data
- Identify and translate data patterns into actionable business insights
- Answer questions by creating experiments, analysing and interpreting datasets
- Understand machine learning and predictive models Here are a few examples of statistics principles you'll need to know to break into the data science field:
- Descriptive statistics - Analyses a dataset to summarise its main

characteristics, like mean and mode

- Inferential statistics - Extrapolates from known data to make predictions or generalisations about a larger population
- Linear regression - Predicts the relationship between an dependent variable and two or more independent variables
- Statistical experiments - Know how to create statistical hypotheses, do A/B testing and other experiments, and form conclusions

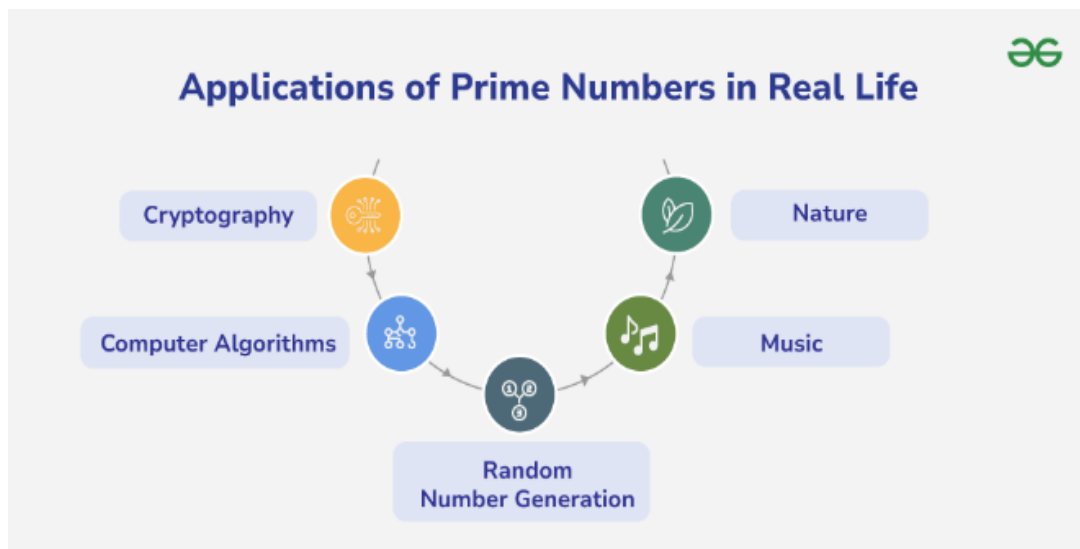
In contrast, probability is the likelihood that an event will occur. Data professionals use this method to analyse risk, forecast trends, and predict the outcomes of business decisions.

Application of Prime Numbers in Real Life

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A prime number is a natural number which is divisible only by 1 and the number itself. Prime numbers are heavily used in computer algorithms, cryptography and hashing. Below are the details.



Cryptograph

RSA Encryption, The RSA Algorithm is said to secure 90% of the Internet. When you type a URL, you see https. The S here means secure and security is provided based on RSA. The RSA algorithm is based on the fact that prime factorization of large numbers is difficult to do. Suppose we multiply two prime numbers together to make a big number. This big number will act as our secret code, which is known as the Public Key. We keep one of the original prime numbers a secret (which is our private key). When our friend wants to send a secret message, they use our Public Key to scramble it. Only our private key can unscramble the message which makes our message secure.

Cryptography, the science of secure communication, depends upon prime numbers to ensure safe internet communication, for secure encryption, key generation, and cryptographic protocols.


Internet Security

Prime numbers are used when we set a strong password for our social media or email account. They are used by encryption techniques to create distinct keys, which keep the

private data safe from hackers. They are also critical to guarantee the security of online transactions and data privacy.

Prime Numbers are used in different fields to provide us Internet security.

Digital Signatures

Prime numbers also play a role in creating digital signatures. When we sign our name on a letter, digital signatures use prime numbers to create a unique signature  for each. This ensures the safe and secure way to send and receive authentic signatures over internet.

Key Exchange

Cryptography also helps us in secure and safe web browsing with the help of a process known as Key Exchange. When we visit any secure website (like online banking site, government sites, etc), our computer and the website need to agree on a secret code to keep the communication private and safe.

Here, prime numbers are used in this process called key exchange. Prime numbers help us generate these secret passwords securely. This unique property of prime number helps to maintain confidentiality, integrity, and authenticity of our digital communication.

Internet Security

Prime numbers are used when we set a strong password for our social media or email account. They are used by encryption techniques to create distinct keys, which keep the private data safe from hackers. They are also critical to guarantee the security of online transactions and data privacy.

Prime Numbers are used in different fields to provide us Internet security.

Image Compression

Prime numbers plays an important role in image compression algorithms used in Radiology or medical imaging. Medical images, like X-rays or MRIs, are really big files, which makes them hard to store or send. Prime numbers help in compressing these files without losing any important and minute diagnostic information.

Signal Processing



Medical imaging techniques such as MRI and CT scans generate signals that undergo digital processing before being reconstructed into images. Prime numbers are used in signal processing algorithms for various tasks such as noise reduction, and filtering these images. Sometimes, these images have some unwanted lines which are removed by filters created with the help of prime numbers.

However Prime Numbers are not directly involved in the process of medical imaging but they influence this process with mathematical principles and algorithms to make image processing and its analysis more efficient.

Mathematical Research

Prime numbers have a rich area of study in the field of mathematics with numerous applications and implications. Here are some ways in which prime numbers are used in mathematics research:

Prime Factorization Algorithms

Prime factorization is a process of breaking composite numbers into their prime factors. This is a fundamental problem in number theory with huge practical applications in Cryptography, Computer Science, and Engineering.

Mathematics research in this area focuses on developing efficient algorithms for prime factorization, improving existing methods, and exploring the complexity of factorization problems.

Cracking Codes

Prime numbers are used to keep our online information safe, like when we shop online or create and use our social-media handles. Mathematicians work on ways to make these codes really hard to crack. This helps to maintain our security and safety on

internet. In mathematical research, mathematicians studies how prime numbers can be used to create strong codes to keep our personal information secure.

Making Algorithms Faster

Prime numbers are used in algorithms that help computers to work more efficiently. Researchers work on ways to use prime numbers to solve problems quickly, like breaking down big numbers into their smaller parts or finding the factors of a number. This helps to improve the efficiency of computers and make them faster. Apart from these, prime numbers are used in various other mathematical concepts such as Algebraic Number Theory, Analytic Number Theory, Finding Patterns, etc.

Error Detection and Correction

Prime numbers are also utilized in error-detection and error-correcting codes for data transmission and storage systems. The use of prime numbers in error detection and correction is described below:

Error-Correcting Codes

Error-correcting codes are mathematical algorithms that are created to encode data in such a way that errors can be easily detected during transmission or storage.

Prime numbers are mainly used in the design of these error-correcting codes, such as Reed-Solomon codes and BCH (Bose-Chaudhuri-Hocquenghem) codes. These codes depends upon algebraic structures where prime numbers are used to determine the field size of these structures. By properties of prime numbers, error-correcting codes can detect and correct errors efficiently, maintaining the transmitted data more reliable and accurate.

WORKS OF ARYABHATTA'S IN MATHEMATICS

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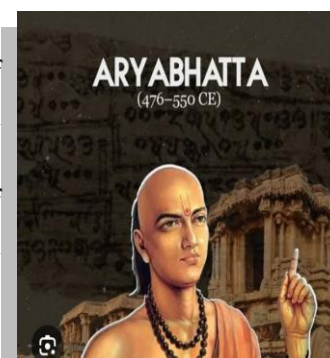
Introduction:-

Aryabhata was an important figure in traditional Indian mathematics and astronomy from the classical period forward, India has produced a long line of visionary mathematicians known as mathematicians of vision. Modern astrophysics and mathematics may trace their roots back to the research and writings linked with him, which were decades ahead of their time when they were first published.

Biography:-

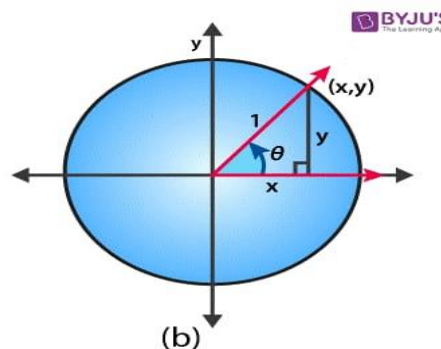
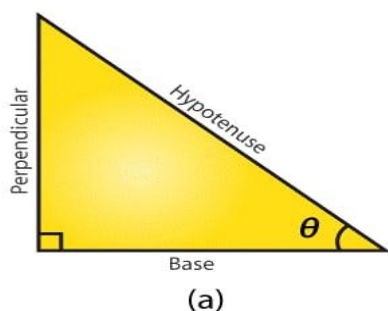
ARYABHATTA (476-550CE)

Aryabhata was born in 476 AD is recognized as the birthplace of the Indian philosopher. Aryabhata was born in Patliputra which is present time is in Patna, Bihar state. It is believed that he completed his studies in Kusum-pura. Aryabhata was the head of Kusum-pura institution. He was also the head of the Nalanda University, Bihar, because the university was located near Patliputra and housed astronomical observations. He died at the age of 74 in 550 CE after a long career as mathematician, astronomer and scientist. In Kusum-pura, Patliputra, it was thought that he spent most of his times.

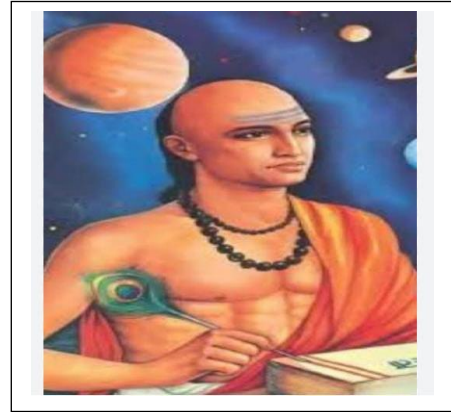


Contributions of ARYABHATTA in Mathematics:-

Aryabhata made several contributions to mathematics inventions and theories. Due to this significant contribution and achievement in mathematics, he is also called the king of Indian mathematics, some of the important discoveries he made in the mathematics field are:



- The place value system and zero
- Trigonometry
- Algebra
- Astronomy
- Approximation of π
- Indeterminate equations



➤ Contributions of Aryabhata in Astronomy:-

Besides mathematics Aryabhata also made several impactful discoveries and inventions in astronomy. Scientists made several discoveries based on his discoveries such as that planets and moon in the solar system are lightened by sunlight only. He gave the theory that Earth rotates on its axis only. Some of the Aryabhata's significant contributions to Astronomy include:

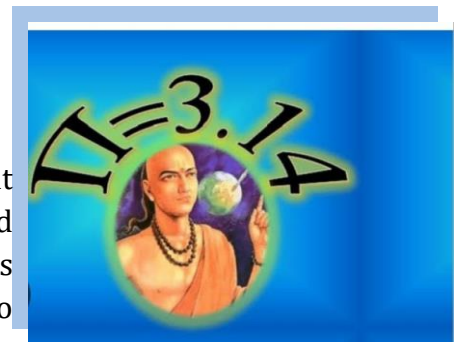
- Solar system motion
- Sidereal periods
- Ellipses
- Heliocentrism

Aryabhata also wrote several books about his discoveries and pieces of work in mathematics and astronomy. Many of his books were lost and never discovered. Some of the well-known books written by Aryabhata are:-

- Aryabhatiya
- Rishab's Good Theory of Indian
- Dash Geetika
- Arya Siddhanta

Contribution in the approximation of π (π):-

Aryabhata is among the great mathematicians who brought new deductions and theories in mathematics and astronomy. His contribution to mathematics is unmatched and cannot be ignored as he was the one who deduced the approximate value of π , which he found it to be 3.14. He also derived the correct formulas for calculating the areas of triangles and circles. He also played a very important role in the formation of the table of sines.



Role in the placed value system :-

He also played a very major role in determining age place value system and discovering the zero . He also worked on the summation series of square roots and cube roots .He also regarded as the first to use zero in the place value system . He also calculated the sidereal rotation , which is the rotation of the earth in relation to the fixed stars . His theories and deductions formed the base of the trigonometry and algebra . Aryabhata was the head of Kusum-pura.

Aryabhata Legacy

Aryabhata died in 550 CE in Patli-putra only. The contributions made by Aryabhata are still used in today's times. Go through Aryabhata's work that is still practised.

- Aryabhata's astronomical calculating methods are used in the Islamic world to calculate dates for calendars.
- Trigonometric tables are used to compute numerous Arabic astronomy tables.
- Aryabhata's definitions of cosine, sine, versine and inverse sine impacted the development of trigonometry mathematics. Moreover, he was also the first to give sine and versine ($1 - \cos x$) tables in 3.75° intervals from 0° to 90° , with 4 decimal places of precision.
- The contemporary terms 'sine' and 'cosine' are mistranslations of Aryabhata's phrases jy and koji.



ROLE OF MATHEMATICS IN MACHINE LEARNING

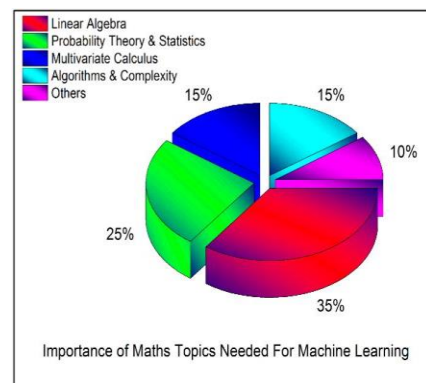
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INTRODUCTION

Mathematics is the foundational backbone of machine learning (ML), providing the theoretical framework and essential tools for developing algorithms and models that enable learning from data, making predictions, and minimizing errors.

Many people are aiming to transfer to the AI/ML/Data Science area these days, which is quite encouraging and in line with the world's changing speed. However, these individuals are perplexed by questions such as: I want To be a machine learning expert without having to learn a lot of math. Is that possible? What role does Mathematics play in data science and AI/ML? As previously said, there are numerous libraries available to Conduct various machine learning tasks, making it simple to ignore the mathematical aspects of the topic. Various issues, such as computer games, self-driving automobiles, and object recognition, are difficult to tackle With traditional programming methods. Machine learning is one approach to tell computers how to learn from Data. Machine Learning is used to assist Amazon propose products to you, YouTube recommend videos, and Spam mail be classified, among other things. To achieve this, we use a combination of mathematics and a lot of Programming. The goal of machine learning is to create algorithms that can learn from data and generate Predictions [1]. Machine learning is predicated on mathematical foundations. Mathematics is required to Complete the Data Science project and to solve the Deep Learning use cases. Mathematics clarifies the basic Principle of the algorithms and explains why one is superior to the other. You can develop models even if you Don't understand the logic behind how algorithms function, but wait... What would you do if you didn't know Which one was best and when to utilise it? To work as a data scientist, you must be familiar with the Mathematics that underpin machine learning techniques. It's unavoidable. Every recruiter and seasoned Machine learning specialist would attest to the fact that it is a crucial aspect of a data scientist's job.

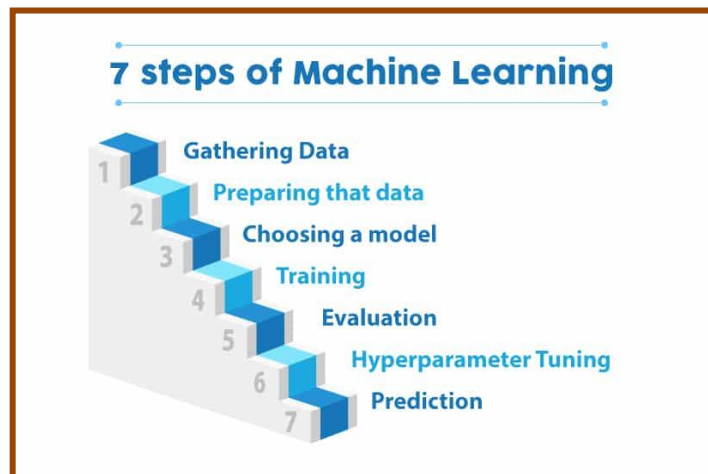


Machine learning process

STEP 1: Collecting Data:

Machines, as you may know, learn from the data you provide them with. It's critical to get trustworthy data so that your machine learning model can uncover the right trends. The accuracy of your model is determined by the quality of the data you provide the machine. If your data is faulty or obsolete,

you'll get inaccurate results or forecasts that aren't useful. Make sure you utilize data from a reputable source, as it will have a direct impact on the model's conclusion. Good data is meaningful, has few missing and duplicated numbers, and accurately represents the many subcategories/classes[17].



STEP 2: Data Pre-Processing:

Putting all your info together and randomizing it. This ensures that data is dispersed uniformly and that the ordering has no effect on the learning process. Unwanted data, missing values, rows, and columns, duplicate values, data type conversion, and so on are all removed from the data [18]. It's possible that you'll need to rearrange the dataset and alter the rows and columns, as well as the indexes of rows and columns. Visualize the data to see how it's organized and to see the connections between the various variables and classes. Creating two sets of cleansed data: a training set and a testing set. The training set is the one from which your model learns. A testing set is used to assess your model's correctness after it has been trained.

STEP 3: Choosing a Model

After performing a machine learning algorithm on the obtained data, a machine learning model selects the output. It is critical to select a model that is appropriate for the work at hand. Over time, scientists and engineers have built a variety of models for diverse tasks such as speech recognition, picture recognition, prediction, and so on. Aside from that, you must determine if your model is best suited for numerical or categorical data and make the appropriate choice[19].

STEP 4: Training the Model

The most crucial phase in machine learning is training. To detect patterns and create predictions, you give the prepared data to your machine learning model during

training[20]. Consequently, the model learns from the Data and can complete the goal assigned. The model improves in predicting over time as it is trained.

STEP 5: Evaluating the Model

After you've trained your model, you'll want to see how it's doing. This is accomplished by putting the model to The test on previously unknown data. The testing set that you split our data into before is the unseen data Utilized. If you test on the same data that was used for training, you won't receive an accurate result since the Model is already familiar with the data and recognizes the same patterns it did before. This will provide you With a disproportionately high level of precision. When applied on testing data, you can receive a precise Estimate of how your model will perform and how fast it will run[21].

STEP 6: Parameter Tuning

Examine whether your model's accuracy can be enhanced in any manner once you've constructed and tested it. This is accomplished by fine-tuning the parameters in your model. Parameters are the variables in the model That are set by the programmer. The accuracy will be at its highest for a certain value of your parameter[22]. Finding these settings is referred to as parameter tweaking.

STEP 7: Making Predictions

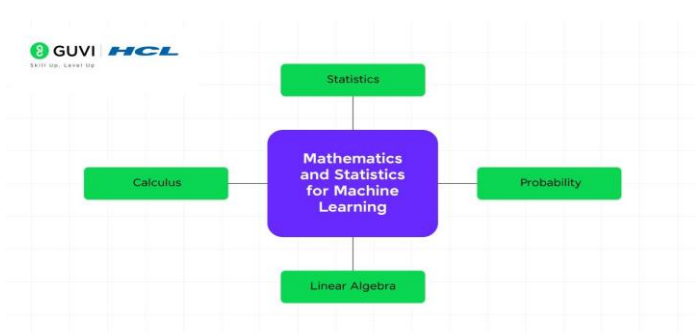
Finally, you'll be able to generate accurate predictions using your model on previously unknown data.

MATHEMATICS USED IN MACHINE LEARNING

Most of our real-world business problems are solved using four pillars of Machine Learning. These pillars are also used in many Machine Learning techniques. They really are.

- ☐ Statistics
- ☐ Probability
- ☐ Linear Algebra
- ☐ Calculus

Machine learning is all about dealing with data. We collect data from organizations or from any repositories like Kaggle, UCI etc., and perform various operations on the dataset like cleaning and processing



the data, visualizing and predicting the output of the data. For all the operations we perform on data, there is one common foundation that helps us achieve all of this through computation—and that is Math

1. **LINEAR ALGEBRA:**

Vectors and Matrices: Machine learning algorithms often represent data and model parameters as vectors and matrices, making linear algebra essential for data manipulation and transformation.

Eigenvalues and Eigenvectors: These concepts are used in dimensionality reduction techniques like Principal Component Analysis (PCA).

2. **CALCULUS:**

Derivatives and Gradients: Calculus is used to optimize machine learning models by finding the optimal parameters that minimize a cost function (or maximize a likelihood function).

Multivariate Calculus: Essential for understanding and working with models that have multiple input variables.

3. **STATISTICS AND PROBABILITY:**

Probability Distributions: Understanding probability distributions (e.g., normal, binomial) is crucial for modeling data and making predictions.

Descriptive Statistics: Used to summarize and analyze data, providing insights into patterns and trends.

Bayesian Statistics: Used in Bayesian machine learning models, allowing for incorporating prior knowledge and updating beliefs based on new evidence.

Regression Analysis: Used to model the relationship between variables and make predictions.

INTRODUCTION TO MATHEMATICAL BIOLOGY

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➤ Definition and Scope of Mathematical Biology:

Mathematical biology is an interdisciplinary field that combines mathematical and computational techniques to analyze and model biological systems. It aims to understand the complex interactions within biological systems, from the molecular to the ecosystem level, using mathematical and computational tools. Mathematical biology draws on techniques from mathematics, statistics, computer science, and engineering to develop new insights and understanding of biological systems

➤ History of Mathematical Biology:

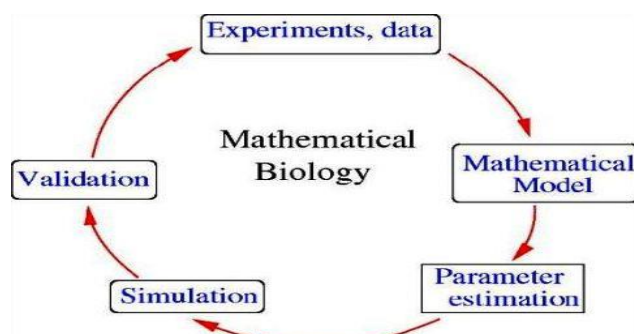
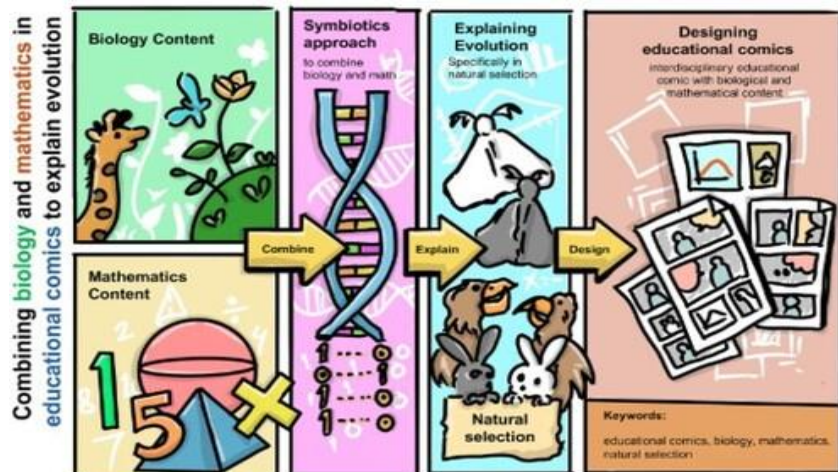
The field of mathematical biology has its roots in the early 20th century, when mathematicians and biologists began collaborating to understand the dynamics of population

growth and epidemiology. One of the earliest examples of mathematical biology is the work of Ronald Ross, who used mathematical models to understand the spread of malaria in the early 20th century. Since then, mathematical biology has expanded to encompass a wide range of topics, including molecular biology, ecology, evolution, and systems biology.

➤ Key Concepts and Techniques:

Mathematical biology employs a range of mathematical and computational techniques, including:

1. Dynamical systems: to model the behavior of biological systems over time



2. Probability and statistics: to analyze and interpret biological data
3. Optimization techniques: to understand the optimal behavior of biological systems
4. Network analysis: to study the complex interactions within biological systems
5. Computational modeling: to simulate the behavior of biological systems

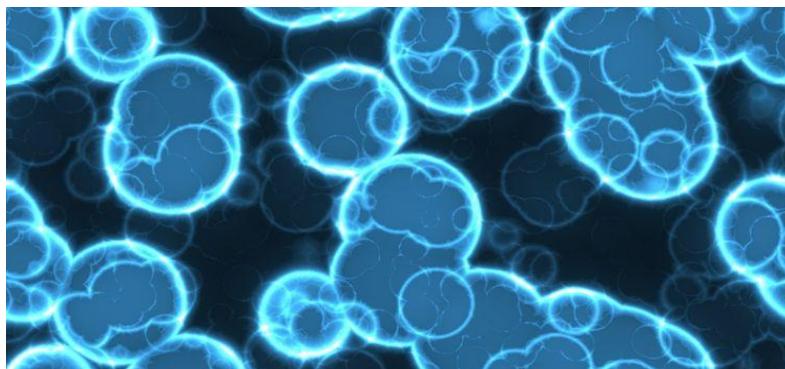
➤ **Applications of Mathematical Biology:**

Mathematical biology has a wide range of applications, including:

1. Epidemiology: to understand the spread of diseases and develop control strategies
2. Ecology: to study the dynamics of populations and ecosystems
3. Systems biology: to understand the complex interactions within biological systems
4. Personalized medicine: to develop tailored treatments for individual patients
5. Synthetic biology: to design new biological systems.

➤ **Case Studies:**

1. Modeling the Spread of Infectious Diseases: Mathematical biologists have developed models to understand the spread of infectious diseases, such as SARS and Ebola. These models have been used to develop control strategies and predict the impact of interventions.
2. Understanding the Dynamics of Cancer: Mathematical biologists have developed models to understand the dynamics of cancer growth and treatment response. These models have been used to develop new treatment strategies and predict patient outcomes.
3. Modeling the Behavior of Complex Biological Systems: Mathematical biologists have developed models to understand the behavior of complex biological systems, such as gene regulatory networks and metabolic pathways. These models have been used to understand the underlying mechanisms of biological systems and develop new therapeutic strategies.

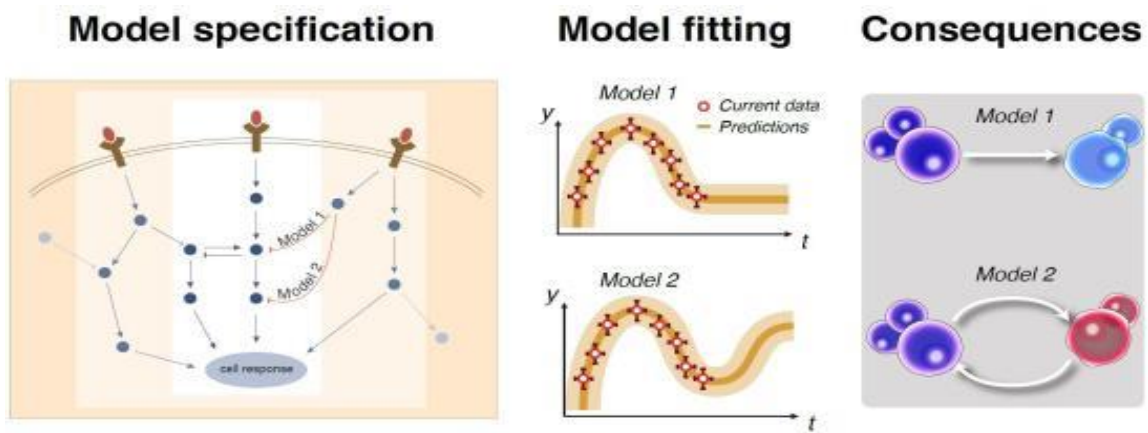


Mathematical biology and medicine

➤ Mathematical Modeling in Biology:

Mathematical modeling is a key tool in mathematical biology, allowing researchers to simulate and predict the behavior of biological systems. There are several types of mathematical models used in biology, including:

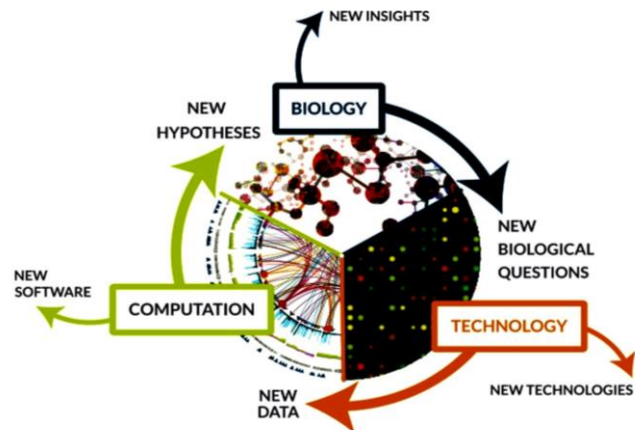
1. Deterministic models: which assume that the behavior of the system is entirely determined by its initial conditions and parameters.
2. Stochastic models: which assume that the behavior of the system is subject to random fluctuations.
3. Hybrid models: which combine elements of deterministic and stochastic models.



➤ Computational Tools and Techniques:

Mathematical biologists use a range of computational tools and techniques, including:

1. Programming languages: such as Python, R, and MATLAB.
2. Software packages: such as COMSOL, BioUML, and CellDesigner.
3. Data analysis and visualization tools: such as Excel, Tableau, and Power BI.



➤ Future Directions:

Mathematical biology is a rapidly evolving field, with new techniques and applications emerging continuously. Some potential future directions include:

1. Integrating machine learning and artificial intelligence: to analyze large biological datasets and develop new predictive models.

2. Developing multiscale models: to bridge the gaps between different biological scales, from molecules to ecosystems.
3. Applying mathematical biology to synthetic biology: to design new biological systems and develop new therapeutic strategies.
4. Developing new mathematical and computational techniques: to analyze and model complex biological systems.

➤ **Conclusion:**

Mathematical biology is a rapidly evolving field that combines mathematical and computational techniques to analyze and model biological systems. It has a wide range of applications, from understanding the spread of infectious diseases to developing new therapeutic strategies. As the field continues to evolve, we can expect to see new techniques and applications emerge, leading to a deeper understanding of complex biological systems and the development of new treatments and therapies.

THE MATH BEHIND EPIDEMICS

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M.Sc. 2nd Sem.



Introduction

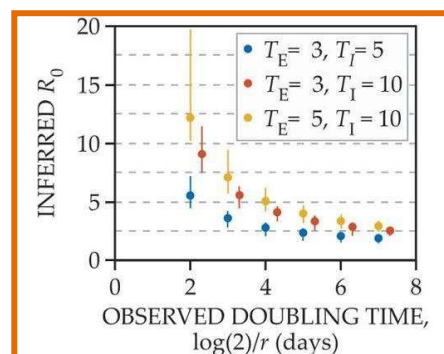
A few simple metrics characterize outbreaks like COVID-19, but calculating them correctly is surprisingly tricky. This article introduces the essential mathematical quantities that characterize an outbreak, summarizes how scientists calculate those numbers, and clarifies the nuances in interpreting them.



For COVID19, estimates of those quantities are being shared, debated, and updated daily. Physicists are used to distilling real-world complexity into meaningful, parsimonious models, and they can serve as allies in communicating those ideas to the public.

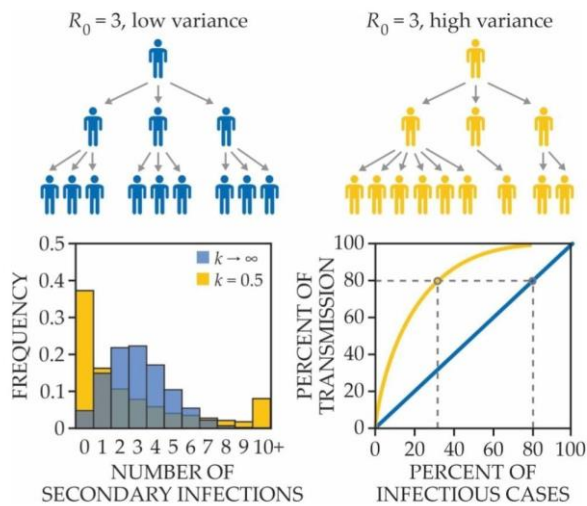
Quantifying transmission dynamics

A disease's basic reproductive number R_0 describes the average number of secondary infections generated by a single infected individual introduced into a susceptible population. For an epidemic to take off, R_0 must be greater than 1. An epidemic will tend to slow if the fraction f of the population that's protected from infection is sufficiently large: $f > 1 - 1/R_0$



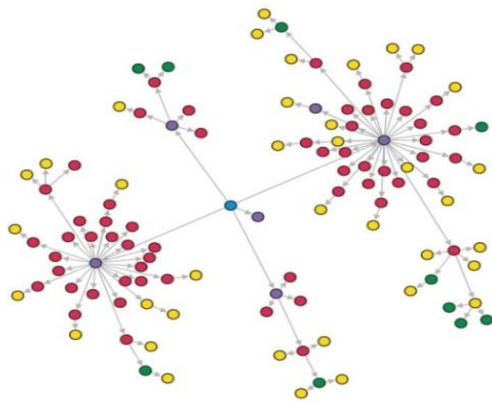
The variance in secondary infections can be large

and can lead to super spreading events. The number of secondary infections is often summarized by a negative binomial distribution, $P(x;R_0,k) = \binom{k+x-1}{x} p^k (1-p)^x$: With mean R_0 , where k parameterizes the dispersion of secondary infections, $p = (1 + R_0/k)^{-1}$, and Γ is the gamma function. If all individuals have the same intrinsic infectiousness-that is, the variance is low (blue scenario on the right) then the number of secondary infections is expected to have a Poisson



distribution ($k \rightarrow \infty$). If heterogeneous, the distribution is said to be over dispersed and has a lower k . Over dispersion implies that a small number of individuals are responsible for a large percentage of secondary infections (dotted lines), whereas most others infect no one, which causes infection chains to go extinct. For COVID-19, a few studies have estimated $k \approx 0.5$ (yellow at right), albeit with high uncertainty.

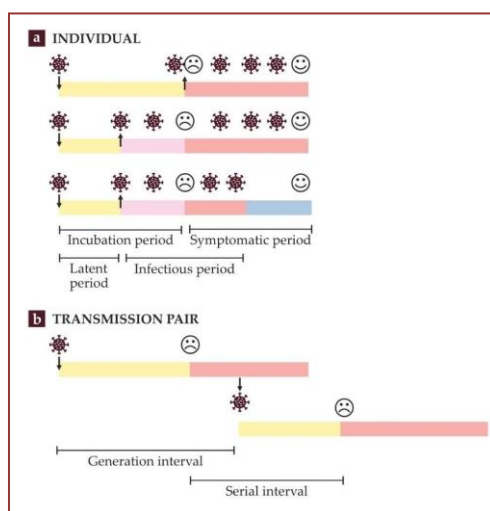
Estimating R_0 directly is difficult. Instead, its value is usually inferred from the disease's exponential growth rate r early in the epidemic and from the infection's time



scale. For example, if the average duration of the latent and infectious periods (T_E and T_I , respectively) are known and one assumes that the periods have exponential distributions, then $R_0 = (1 + rT_E)(1 + rT_I)$ (dots on the lower graph). Other distribution shapes lead to different estimates for R_0 (error bars). Country-level epidemic growth rates in the range of 0.10.4 per day have been observed for COVID-19, which corresponds to doubling times of 2-8 days. Estimates of R_0 have

generally been between 2 and 3, although they are sometimes much higher depending on the setting observed and the assumptions about the transmission intervals.

In contact-tracing studies, as soon as an individual is diagnosed, public health professionals track down anyone that person might have contacted during his or her infectious period and test them for the disease; researchers use the data to estimate for a single generation of infection.

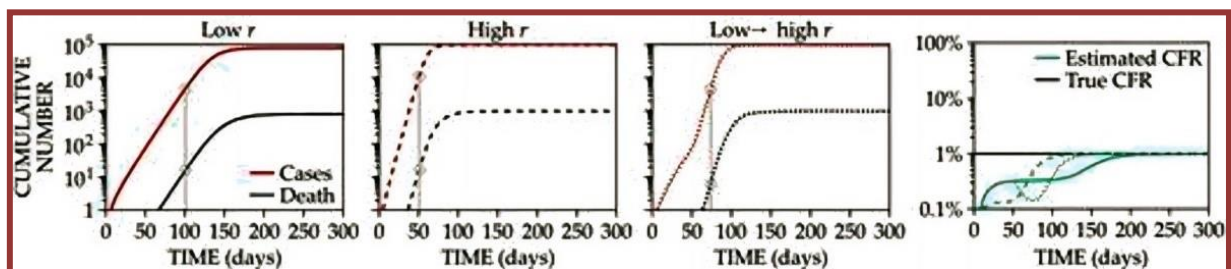


Time scale of infection

Exponential growth in the number of infections is a defining feature of epidemics early in their course. Estimates of the growth rate r , or alternatively the doubling time $T_2 = \log(2)/r$, can inform short-term projections of the epidemic. In general, those formulas require knowing how long a typical individual is infectious and the delay between when someone is infected and when they become infectious, known as the latent period. A high

observed exponential growth rate of infection implies a high R_0 if either the latent period or infectious period is long, whereas it could imply a much smaller value if both those intervals are short.

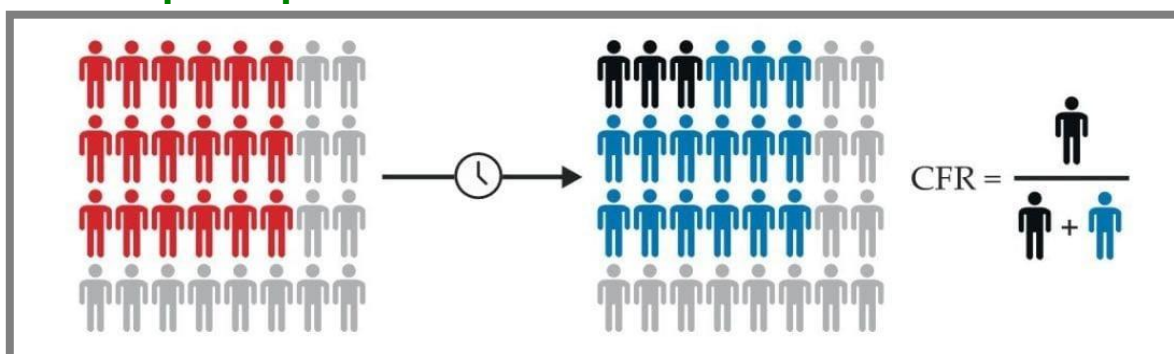
Estimating the risk of dying during an epidemic



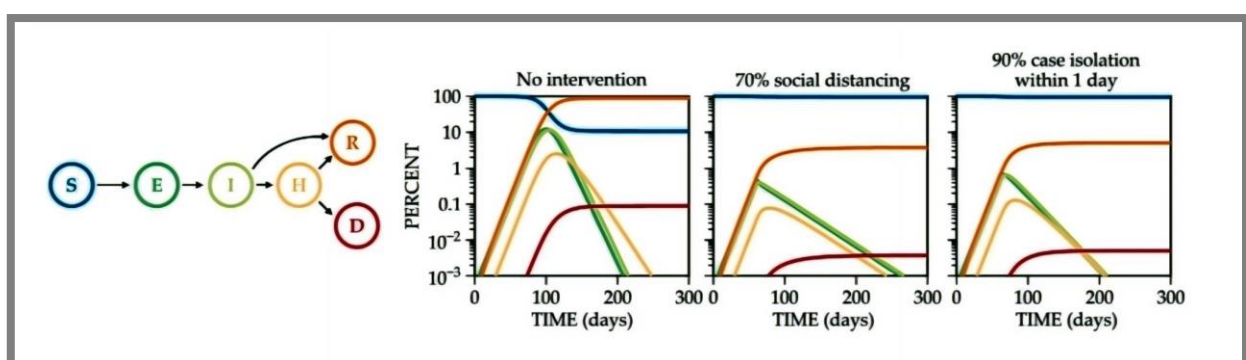
Epidemiologists use a disease's case fatality risk (CFR) to describe the percentage of individuals confirmed to be infected (red, top right) who will eventually die of the disease (black) rather than recover (blue). The true CFR can be accurately established only by following a cohort of infected individuals until their final outcome is observed.

In the simple infection model shown here, individuals are only infectious for about five days, but it may take an additional two weeks for them to die. The true CFR is 1%, which is dramatically underestimated by the ratio of deaths to cases early in the epidemic. In real data, the ratio can further be confounded by underreporting or reporting delays.

From description to prediction

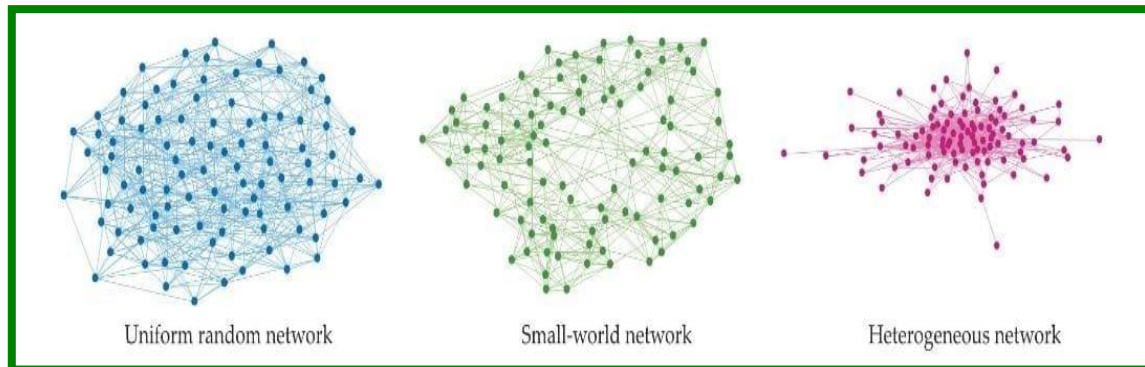


The model tracks changes in the number of individuals in each state, usually with differential equations or discrete or continuous stochastic processes. The equations are inherently nonlinear because pairwise interactions between susceptible and infectious individuals generate new infections.



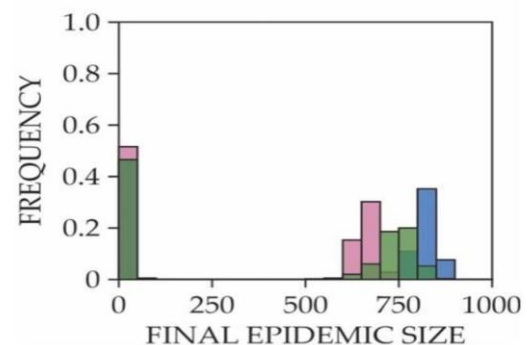
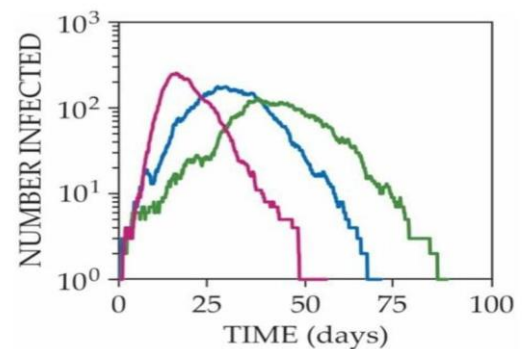
Simulating infection spread in networks

Human contacts are not random or uniformly distributed. They can be described by a contact network that determines which transmission paths are possible if an infection is introduced into the population. The structure of the network can heavily influence the extent of disease spread.



Epidemic growth is fastest in the heterogeneous networks and boosted by highly connected super spreaders. It's slowest in the small-world network, where the high degree of interconnectedness limits the susceptible contacts seen by an infected individual. The final epidemic size-the percentage of recovered individuals when the infection eventually dies out-varies across simulations because of stochastic effects, but it is generally highest in the uniform network and lowest in the heterogeneous network.

Mathematical analysis and modelling are key tools in the study of infectious diseases and have been critical in our response to the COVID-19 pandemic. Estimating even seemingly simple metrics-Ro, the CFR, and the incubation and infectious periods, among others-requires strict attention to nuances in the data and careful formulation of mathematical relationships. When designing complex models of epidemic dynamics, modelers make trade-offs between keeping things simple enough to facilitate understanding and realistic enough to make accurate forecasts. Getting the numbers right is always a priority for scientists. During a public health crisis, the stakes are higher than ever.



FUZZY LOGIC IN AI

INTRODUCTION:-

Fuzzy Logic (FL) is a method of reasoning that **resembles human reasoning**. This approach is similar to how human perform decision making and it involves all intermediate possibilities between **YES** and **NO**.

ARCHITECTURE:-

Its architecture contains four parts

- RULE BASE
- FUZZIFICATION
- INFERENCE ENGINE
- DEFUZZIFICATION

MEMBERSHIP FUNCTION:-

The membership function is a **graph** that defines how each point in the **input space** is mapped to membership value between 0 and 1. It allows you to **quantify linguistic terms** and represent a fuzzy set graphically. A membership function for a fuzzy set A on the universe of discourse X is defined as $\mu_A: X \rightarrow [0,1]$

It quantifies the degree of membership of the element in X to the fuzzy set A.

- **x-axis** represents the universe of discourse.
- **y-axis** represents the degrees of membership in the [0, 1] interval.

There can be multiple membership functions applicable to fuzzifying a numerical value. Simple membership functions are used as the complex functions do not add precision in the output.

The membership functions for **LP, MP, S, MN, and LN** are:



Khushi Raikopand
M.Sc. 2nd Sem.

The triangular membership function shapes are most common among various other membership function shapes. Here, the input to 5-level fuzzifier varies from **-10 volts to +10 volts**. Hence the corresponding output also changes.

FUZZY LOGIC IN AI:-

Fuzzy logic in artificial intelligence (AI) is a way of reasoning that uses degrees of truth to imitate human thought. It's used to solve problems that involve uncertainty, ambiguity, or imprecise data.

FUZZY LOGIC 'S IMPORTANCE IN AI:-

Fuzzy logic is crucial in AI for handling uncertainty, ambiguity, and imprecise data, enabling systems to make decisions more like humans with several characteristics that mimic our decision-taking behavior:

- Works with imprecise inputs
- Provides smoother transitions between states
- Enables nuanced reasoning in complex environments
-

APPLICATION OF FUZZY LOGIC:-

- **Aerospace**

Altitude control of spacecraft, satellite altitude control and mixture regulation in aircraft de-icing vehicles.

- **Automotive**

Trainable fuzzy systems for idle speed control, shift scheduling method for automatic transmission, intelligent highway systems, traffic control, improving efficiency of automatic transmissions.

- **Business**

Decision-making support systems, personnel evaluation in a large company.

- **Chemical Industry**

Control of pH, drying, chemical distillation processes, polymer extrusion production, a coke oven gas cooling plant.

The fuzzy logic checks for the extent of dirt and grease, the amount of soap and water to add, direction of spin, and so on.

The machine rebalances washing load to ensure correct spinning. Else, it reduces spinning speed if an imbalance is detected. Even distribution of washing load reduces spinning noise. Neuro fuzzy logic incorporates optical sensors to sense the dirt in water and a fabric sensor to detect the type of fabric and accordingly adjust wash cycle.

FUZZY LOGIC IN A WASHING MACHINE :-

Fuzzy logic washing machines are gaining popularity. These machines offer the advantages of performance, productivity, simplicity, productivity, and less cost. Sensors continually monitor varying conditions inside the machine and accordingly adjust operations for the best wash results. As there is no standard for fuzzy logic, different machines perform in different manners.

Fuzzy logic controls the washing process, water intake, water temperature, wash time, rinse performance, and spin speed optimizes the life span of the washing machine.

Machines even learn from past experience, memorizing parameters and adjusting them to minimize running costs.

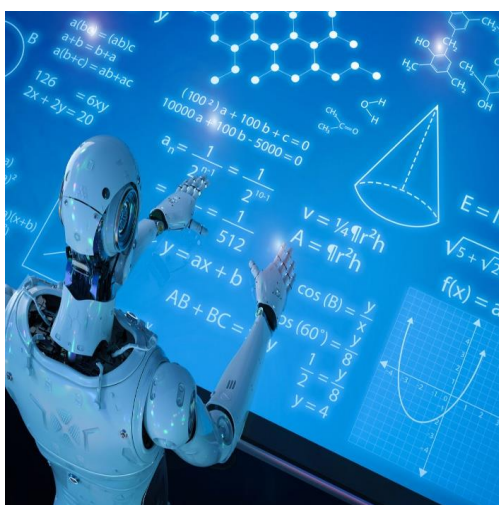
Most fuzzy logic machines feature 'one touch control'. Equipped with energy saving features.

Why Is Mathematics Vital To Thrive In Your Ai Career

Mathematics plays a crucial role in the development and implementation of Artificial Intelligence (AI) systems.

AI relies on mathematical concepts and algorithms to process and analyze large amounts of data.

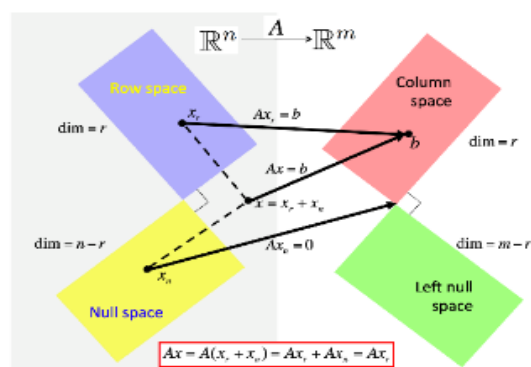
Mathematical models provide a foundation for building intelligent systems and making informed decisions.



LINEAR ALGEBRA IN AI

- Linear algebra is fundamental to AI as it deals with vectors,
- matrices, and linear transformations.
- In AI, linear algebra is used for tasks such as dimensionality reduction, image recognition, and natural language processing.
- Concepts like eigenvectors and eigenvalues are employed to understand patterns and extract meaningful information.

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CALCULUS IN AI

- Calculus is essential for optimization problems in AI, such as finding the minimum or maximum of a function.
- Techniques like gradient descent use calculus to train machine learning models.
- Calculus also helps in understanding the rate of change and modeling dynamic systems in AI.

PROBABILITY AND STATISTICS IN AI

- Probability theory is crucial for dealing with uncertainty in AI systems.

- Statistical techniques, such as hypothesis testing and regression analysis, are used to analyze and interpret data.
- Bayesian networks utilize probability theory to model and reason under uncertainty.

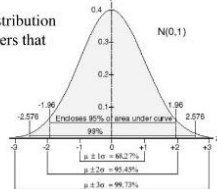
Probability theory

- Probability theory quantifies uncertainty using 'distributions'
- Distributions are the 'models' and they depend on constants and parameters

E.g., in one dimension, the Gaussian or Normal distribution depends on two constants e and π and two parameters that have to be measured, μ and σ

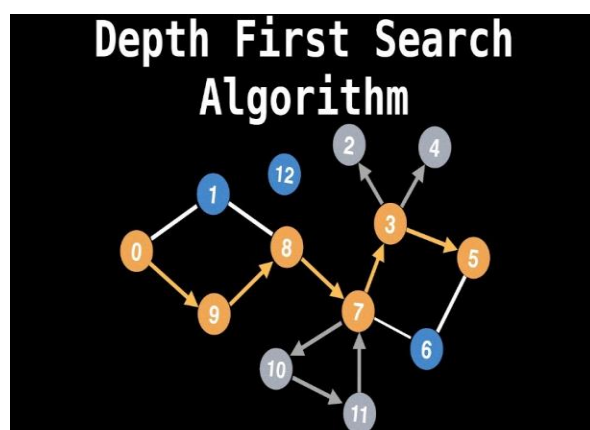
$$P(X|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$$

'X' are the possible datapoints that could come from the distribution. In statistics jargon 'X' is called a random variable



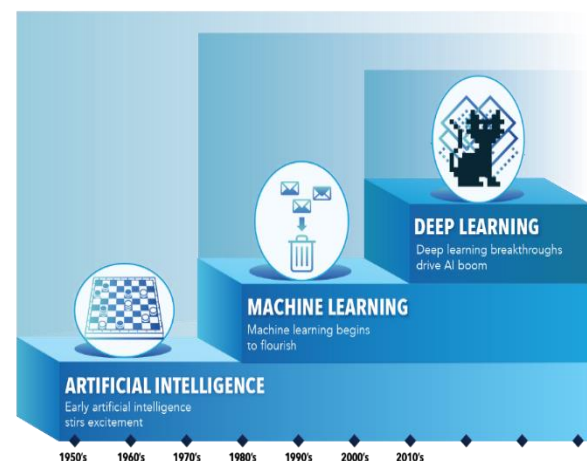
GRAPH THEORY IN AI

- Graph theory provides tools to represent and analyze relationships between entities in AI.
- Algorithms like breadth-first search and depth-first search are used to traverse and explore graphs.
- Graph theory is applied in various AI applications such as social network analysis and recommendation systems.



OPTIMIZATION IN AI

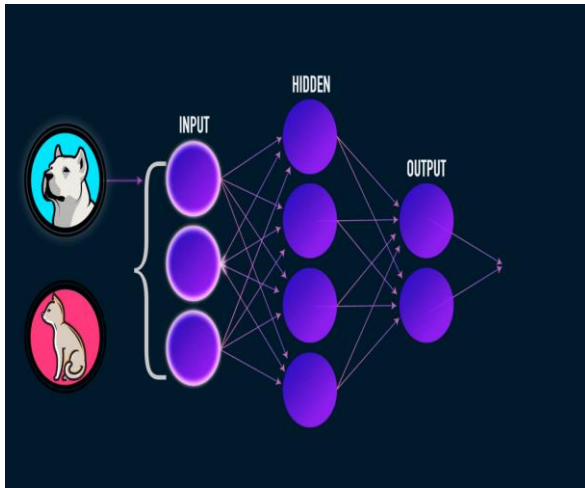
- Optimization is at the core of AI, enabling the selection of the best solutions from a set of possibilities.
- Techniques like linear programming, integer programming, and evolutionary algorithms are used to solve optimization problems in AI.
- Optimization algorithms help in training models, finding optimal paths, and resource allocation in AI systems.



NEURAL NETWORKS AND DEEP LEARNING

- Neural networks, inspired by the structure of the human brain, are used to solve complex problems in AI.
- The mathematics behind neural networks involves linear algebra, calculus, and optimization techniques.
- Deep learning, a subset of neural networks, uses multiple layers to

**extract hierarchical
representations from data.**



REINFORCEMENT LEARNING

- Reinforcement learning is a branch of AI where an agent learns through interaction with an environment.
- Mathematics, particularly Markov decision processes, are used to model and solve reinforcement learning problems.
- Reinforcement learning algorithms aim to maximize a reward signal by learning optimal policies

Triangle Inequality Theorem



Manshi
M.Sc 2nd Sem

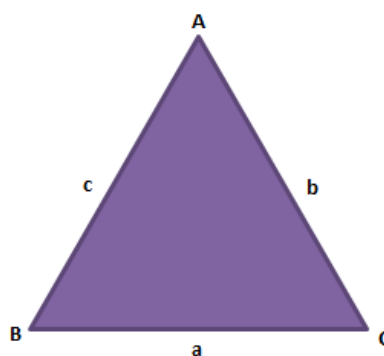
INTRODUCTION:

According to **triangle inequality theorem**, for any given triangle, the sum of two sides of a triangle is always greater than the third side. A polygon bounded by three line-segments is known as the Triangle. It is the smallest possible polygon. A triangle has three sides, three vertices, and three interior angles. The types of triangles are based on its angle measure and length of the sides. The inequality theorem is applicable for all types triangles such as equilateral, isosceles and scalene. Now let us learn this theorem in details with its proof.

Triangle Inequality Theorem Proof

The triangle inequality theorem describes the relationship between the three sides of a triangle. According to this theorem, for any triangle, the sum of lengths of two sides is always greater than the third side. In other words, this theorem specifies that the shortest distance between two distinct points is always a straight line.

Consider a $\triangle ABC$ as shown below, with a , b and c as the side lengths.



The triangle inequality theorem states that:

$$a < b + c,$$

$$b < a + c,$$

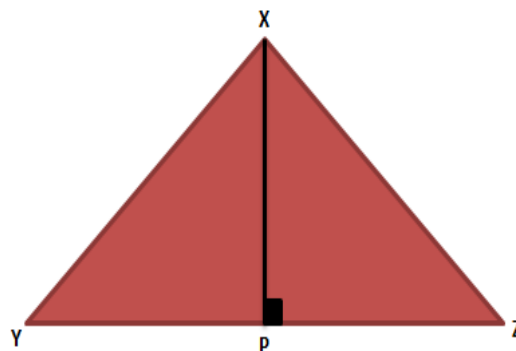
$$c < a + b$$

In any triangle, the shortest distance from any vertex to the opposite side is

the

Perpendicular. In figure below, XP is the shortest line segment from vertex X to side YZ .

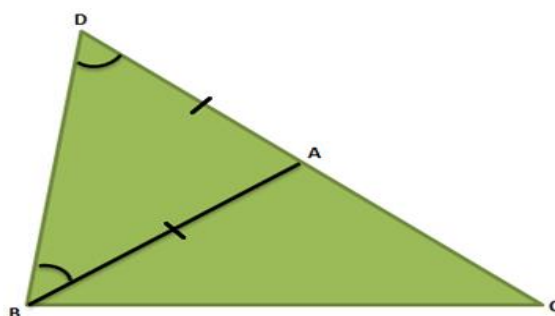
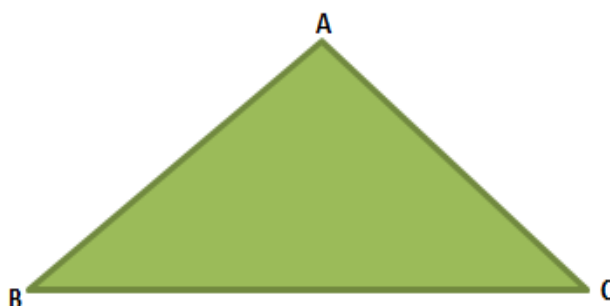
Let us prove the theorem now for a triangle ABC.



Triangle ABC

To Prove: $|BC| < |AB| + |AC|$

Construction: Consider a $\triangle ABC$. Extend the side AC to a point D such that $AD = AB$ as shown in the fig. below.



Proof of triangle inequality theorem

S.No	Statement	Reason
1.	$ CD = AC + AD $	From figure 3
2.	$ CD = AC + AB $	$AB = AD$, $\triangle ADB$ is an isosceles triangle
3.	$\angle DBA < \angle DBC$	Since $\angle DBC = \angle DBA + \angle ABD$
4.	$\angle ADB < \angle DBC$	$\triangle ADB$ is an isosceles triangle and $\angle ADB = \angle DBA$
5.	$ BC < CD $	Side opposite to greater angle is larger
6.	$ BC < AC + AB $	From statements 3 and 4

Thus, we can conclude that the sum of two sides of a triangle is greater than the third side.

Example Problems

If 4cm, 8cm and 2cm are the measures of three lines segment. Can it be used to draw a triangle?

Solution: The triangle is formed by three line segments 4cm, 8cm and 2cm, then it should satisfy the inequality theorem.

Hence, let us check if the sum of two sides is greater than the third side.

$$4 + 8 > 2 \Rightarrow 12 > 2 \Rightarrow \text{True}$$

$$8 + 2 > 4 \Rightarrow 10 > 4 \Rightarrow \text{True}$$

$$4 + 2 > 8 \Rightarrow 6 > 8 \Rightarrow \text{False}$$

Therefore, the sides of the triangle do not satisfy the inequality theorem. So, we cannot construct a triangle with these three line-segments.

Mathematical Cryptography



Mridul Nirmal
M.Sc. 4th Sem.

INTRODUCTION :

Mathematical cryptography is the use of mathematical techniques to ensure the confidentiality, integrity, and authenticity of data, relying on algorithms and mathematical concepts to transform messages into difficult-to-decipher codes.



THE PRINCIPAL GOAL OF CRYPTOGRAPHY, KERCKHOFF'S PRINCIPLE:

The principal goal of cryptography is to allow two people to exchange confidential information, even if they can only communicate via a channel monitored by an adversary. Assume for example that Bob wants to send a message to Alice in such a way that Eve – who reads/listens/spies the communication of Alice and Bob – cannot understand the message (Alice, Bob and Eve are the usual participants

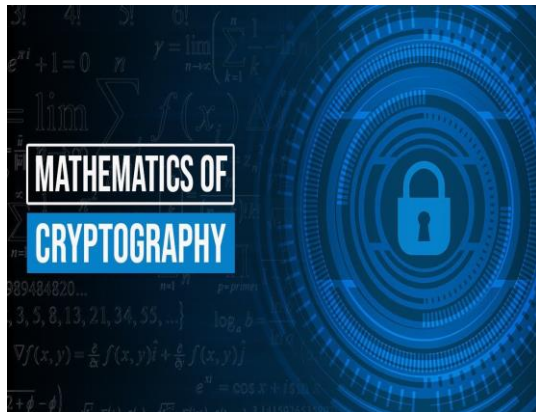
of the cryptographic setup). The scheme of the solution is the following. Bob sends through the communication something else than his original message. Eve can read only this something else. Alice knows how this something else should be understood to get the original message.

CRYPTANALYSIS:

In fact, Caesar's cipher can be easily broken: it is just 26 trials for the shift vector, and this can be done even by a human quite fast. Note that Eve has to try the first few characters in each possible shift, for an incorrect guess, most likely the first few letters will give rise to a gibberish. This is not the case with the simple substitution cipher, the number of possible keys is quite large, $26! > 1026$, which is too many even for a modern PC. However, even such a ciphertext can be revealed relatively easily. The point is that simple substitutions do not alter the characteristic of the underlying natural language, say, English in our case.

So Eve can argue as follows. In a normal English text (which is supposed the message to be), the most frequent characters are 'e', 't', 'a', 'o', 'n'. Even their frequency can be easily computed (you can find online tables telling them, or, by "bare" hand, you can open a long pdf file and count the occurrences of these letters). Also, you can consider the bigrams 'th', 'he', 'an', 're', 'er', and the trigrams 'the', 'and', 'ing'. Now given the ciphertext, if we count the often occurring letters, bigrams and trigrams, we may have reasonable guesses on the

substitution: the point is that, for example, no matter which letter stands for 'e', it will stand for 'e' always, and since 'e' is the most often letter in English, this unknown character will be somewhat often in the cipher (and almost surely the most often, if the cipher is long enough).



The same is true for other frequent letters, and also for bigrams and trigrams. As soon as some parts of the text are revealed, we can figure out further substitutions. After some trial and error, we can recover the text. Although this might seem complicated, it works surprisingly fast, read to see this in action. As a historical note, we remark that cryptanalysis, letter frequency counts were known by Arab scholars in the 14th and the 15th century. The same time, in Italian states, more complicated cryptosystems were used than simple substitution ciphers, which suggests that cryptanalysis via frequency analysis was known there as well.

MATHEMATICS ENTERS CRYPTOGRAPHY:

• Transform texts to numbers,

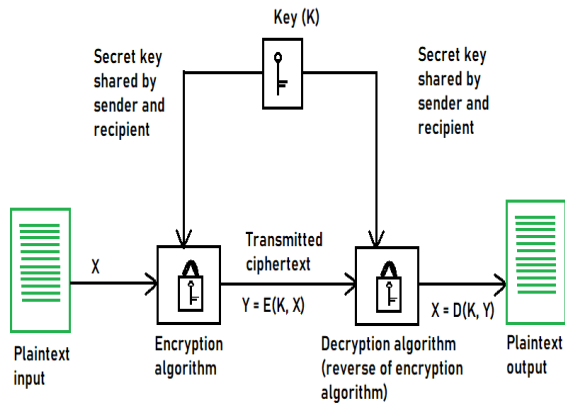
Up to this point, messages and ciphers were both textual objects. This is natural, our communication is textual,

so most messages are simply texts. However, the communication of our computers (which deliver the messages nowadays) are rather based on numbers, and – more importantly – fancy properties of numbers provide us with extremely ingenious cryptosystems. For this reason, given any text, we – more precisely: our softwares – transform it to numbers. Our computers use encoding schemes, e.g. ASCII or UNICODE to convert characters to bytes. A byte consists of eight bits, where a bit is a 0 or a 1 (it is the abbreviation of the binary digit). For example, 01000001 is a byte, and in ASCII, it stands for the character 'A'. This encoding scheme is completely public, its purpose is not to hide information but on the contrary: it guarantees that if we send an e-mail to somebody, our computer transforms it to a number which can be transformed back to the same e-mail by his or her computer. Of course, when Bob's e-mail is encoded in ASCII, and is sent to Alice, when Eve captures it, she is also able to read it. For this reason, Bob, after making the ASCII code, makes the message to a cipher (both are numbers this time). Ideally, when Eve reads the cipher, and decodes its ASCII, she will see a gibberish. Alice, who knows what to do, transforms the cipher back to the ASCII encoded version of the original message, then decodes it.

• The mathematical formulation of symmetric ciphers,

(symmetric cryptosystem). By a symmetric cryptosystem, we mean a 5-tuple (K, M, C, e, d) , where K, M, C are sets,

and $e : K \times M \rightarrow C$, $d : K \times C \rightarrow M$ are functions satisfying that for any $k \in K$ and $m \in M$, $d(k, e(k, m)) = m$.



Here, K is the set of possible keys, M is the set of possible messages, C is the set of possible ciphers. The functions e and d are the encrypting and decrypting algorithms, respectively: for a given key $k \in K$, e computes $c \in C$ from $m \in M$, while d computes $m \in M$ from $c \in C$. In practice, for a fixed $k \in K$, $e(k, \cdot)$ and $d(k, \cdot)$ are often denoted by ek and dk , respectively. In the realization, K, M, C, e, d are known to everyone (including Eve), while the particular $k \in K$ in use is known only to Alice and Bob.

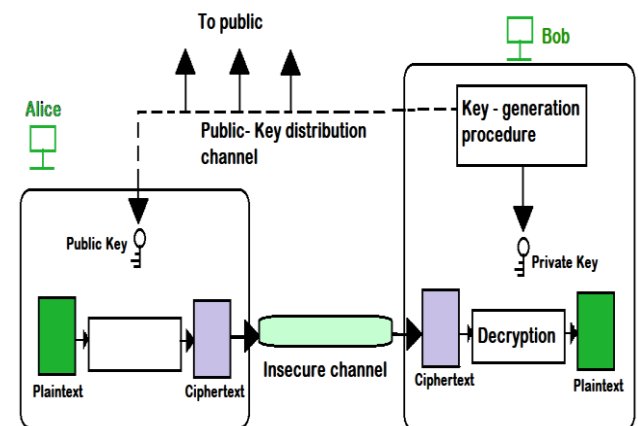
• The mathematical formulation of asymmetric ciphers,

(asymmetric cryptosystem). By an asymmetric cryptosystem, we mean a 7-tuple $(K, K_{pub}, K_{priv}, M, C, e, d)$, where $K, K_{pub}, K_{priv}, M, C$ are sets satisfying $K \subseteq K_{pub} \times K_{priv}$, and $e : K_{pub} \times M \rightarrow C$, $d : K_{priv} \times C \rightarrow M$ are functions satisfying that for any $(k_{pub}, k_{priv}) \in K$ and $m \in M$, $d(k_{priv}, e(k_{pub}, m)) = m$. Here, K is the set of possible keys, M is the set of possible messages, C is the set of possible ciphers.

The functions e and d are the encrypting and decrypting algorithms, respectively: for a given key $(k_{pub}, k_{priv}) \in K$, e computes $c \in C$ from $m \in M$, while d computes $m \in M$ from $c \in C$. In practice, for a fixed $(k_{pub}, k_{priv}) \in K$, $e(k_{pub}, \cdot)$ and $d(k_{priv}, \cdot)$ are often denoted by ek_{pub} and dk_{priv} , respectively.

In the realization, $K_{pub}, K_{priv}, M, C, e, d$ are known to everyone (including Eve). It is only Alice who knows K , and she picks a particular key $(k_{pub}, k_{priv}) \in K$. Now she makes k_{pub} public, and keeps k_{priv} in secret. If Bob wants to send a message $m \in M$ to Alice, then he applies ek_{pub} to m , and sends $ek_{pub}(m)$ to Alice. Now Alice applies dk_{priv} to the incoming cipher, obtaining $dk_{priv}(ek_{pub}(m)) = m$.

The requirements are modified the obvious way.



In this setup, there is nothing special about Bob, he has the same information as Eve has. Yet, if the system works well, Eve is unable to read Bob's messages to Alice. Although this seems more complicated and somewhat artificial, this scheme is commonly used. Think for example of your e-mail address: basically everyone knows it, none of

your friends has a particular role, yet each of them can send you an e-mail which is hidden from anyone else.

COMPUTABILITY:

In this section, we start to investigate the length of computations. Although a precise definition could be as well given using the notion of Turing machine, for our purpose an informal description will be completely sufficient. Assume given an algorithm solving a problem. For example, let the problem be the addition of positive integers (given in base 10), and the algorithm is what we learned in the elementary school. Basically, the algorithm consists of two things: one is the addition table,

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

manipulation, we perform the addition of digits using the table (call them basic additions). If there are two numbers of n digits, then we have to use n basic additions (one at each digit). Also, there might be 1's taken on, at most n times, so the number of basic additions is at most $2n$. This means that given the basic addition table and the school algorithm, they can add up two numbers of n digits in $2n$ many steps, where by one step, we mean one reference to the basic

addition table.

This is the general scheme: we can call anything a step, but that must be fixed once for all; also, we give an algorithm which can use the step as many times as it is needed. Then the input comes, and the algorithm computes the output. The running time (or time cost) of the algorithm (on the given input) is the number of steps used to compute the output.

Assume that the input is the positive integer n . We say that an algorithm computes the output in polynomial time, if the running time can be estimated from above by a polynomial of $\log n$.

Why $\log n$?

Simply because the length of the input is just $\log_{10} n$ (and in other bases, just a constant times $\log_{10} n$), and not n . Similarly, the algorithm is exponential, if it is exponential in $\log n$. If the input consists of more than one numbers, say, $n_1, \dots,$

n_k , then a polynomial algorithm is an algorithm whose running time can be estimated by a polynomial of $\max(\log n_1, \dots, \log n_k)$. From the school, we know that addition, subtraction, multiplication and euclidean division can be performed by polynomial algorithms.

THE XOR CIPHER AND PSEUDORANDOM SEQUENCES:

Assume, the message is a number $0 \leq m < 2^t$, i.e. a binary number on t bits. Now Alice and Bob agree on a binary number k also on t bits. So in this case,
 $M = C = K = \{0 - 1 \text{ sequences of length } t\}$.

Define now the \oplus operation as the bitwise addition, i.e. if $a = \sum_{j=0}^{t-1} a_j 2^j$, $b = \sum_{j=0}^{t-1} b_j 2^j$ (where a_j, b_j 's are binary digits, 0 or 1 each), then let $a \oplus b = \sum_{j=0}^{t-1} c_j 2^j$, where $c_j = 0$ if $a_j = b_j$, and $c_j = 1$ if $a_j \neq b_j$.

Given m and k , let $ek(m) = m \oplus k$. One can easily see that then $dk = ek$:
 $dk(ek(m)) = (m \oplus k) \oplus k = m \oplus (k \oplus k) = m \oplus 0 \dots 0 \mid \{z\} \text{ } t \text{ zeros} = m$.

Since XOR addition can be computed fast, Bob can easily encrypt his message, and Alice can easily decrypt it. However, since Eve does not know k , she essentially has to check all possible k 's between 0 and $2^t - 1$, which is hopeless, if t is large enough.

In some sense, this cryptosystem is as perfect as can be. However, there are a few problems. First, it is vulnerable against a chosen plaintext attack, since even a single pair (m, c) reveals $k = m \oplus c$.

Another problem is that if Alice sends to message with the same key, say $c_1 = ek(m_1)$, $c_2 = ek(m_2)$, then Eve can XOR them to get

$c_1 \oplus c_2 = (m_1 \oplus k) \oplus (m_2 \oplus k) = m_1 \oplus m_2$.
 It is not obvious how Eve can use this information, yet she has managed to get rid of k , and this is something that Alice and Bob would like to avoid.

Third, the key must be as long as the

message is, which requires very long key sequences. And since Alice and Bob would like to use different keys for each encryption, they have to generate many k 's. Their security is the best possible, if they generate random k 's, for example, tossing a 0 - 1 coin t times gives a k , and they can repeat this as many times as they wish. But the setup is that they are at different places, their t -long tosses will almost certainly result differently. How can they solve this problem? Is it possible to securely and efficiently send long messages using only a single short key?

Assume there exists a function $R : K \times N \rightarrow \{0, 1\}$ satisfying the following properties:

- (1) for any $k \in K$, $j \in N$, it is easy to compute $R(k, j)$;
- (2) from any j_1, \dots, j_n and corresponding $R(k, j_1), \dots, R(k, j_n)$, it is hard to figure out k ;
- (3) from any j_1, \dots, j_n and corresponding $R(k, j_1), \dots, R(k, j_n)$, it is hard to guess the value of $R(k, j)$ with better than a 50% chance of success, if $j \notin \{j_1, \dots, j_n\}$.

Now Alice and Bob agree on a key k (a number on 200 bits, say), and they agree that for the first message, they will use $R(k, 1), \dots, R(k, 1000000)$, for the second one, they will use $R(k, 1000001), \dots, R(k, 2000000)$, and so on.

Such functions R are called pseudorandom number generators, however, it is unknown at the moment whether pseudorandom number generators exist.

APPLICATION OF ALGEBRA IN REAL LIFE

PRIYA SINGH THAKUR
M SC 2ND SEM



INTRODUCTION:-

Algebra, a fundamental branch of mathematics, uses symbols and letters (variables) to represent unknown quantities and solve equations, finding applications in various fields and everyday situations.

The word entered the English language in the 16th century, initially referring to the theory of equations, but broadened in the 19th century to encompass various algebraic structures and operations.

EVERYDAY LIFE:-

Budgeting and Finance:-

Algebra helps manage finances by calculating expenses, income, savings, and investments.

Cooking and Recipes:-

Algebraic concepts like ratios and proportions are used to adjust recipes for different serving sizes.

Travel Planning:-

Calculating distance, speed, and time involves using algebraic formulas.

Shopping and Discounts:-

Algebraic equations can help determine the final price after discounts or calculate the best deals.

PROFESSIONAL FIELDS:-

Science and Engineering:-

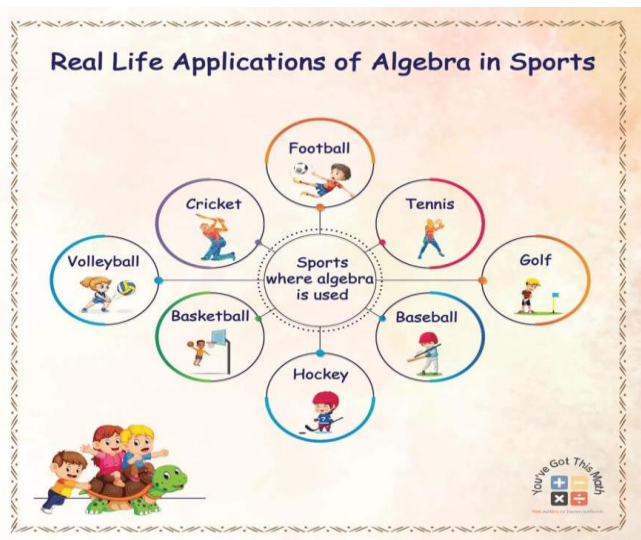
Algebra is crucial for modeling and solving problems in physics, chemistry, and engineering.

Computer Science:-

Algebraic concepts are used in algorithms, data structures, and computer graphics.

Business and Economics:-

Algebra is used for financial analysis, market forecasting, and optimization.



Statistics and Data Analysis:-

Algebra is essential for understanding and manipulating data, calculating probabilities, and drawing conclusions.

Health and Fitness:-

Calculating Body Mass Index (BMI) or tracking progress in weight loss or fitness goals can be done using algebraic equations.

MATHEMATICAL CONCEPTS:-

Solving Equations:-

Algebra provides the tools to solve for unknown variables in equations, which is a fundamental skill in mathematics.

Simplifying Expressions:-

Algebra helps simplify complex expressions, making them easier to understand and manipulate.

Graphing Equations:-

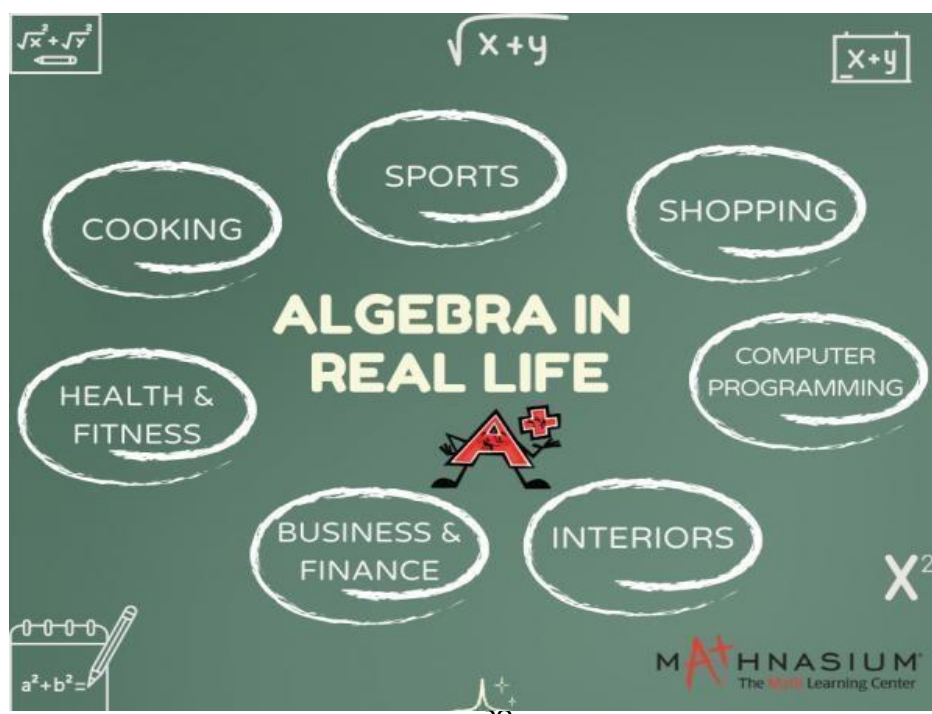
Algebraic equations can be graphed to visualize relationships between variables.

Linear Algebra:-

A branch of algebra that deals with vectors, matrices, and linear transformations, used in many fields like computer graphics and machine learning.

Abstract Algebra:-

A more advanced area of algebra that studies algebraic structures like groups, rings, and field.



ANCIENT INDIAN MATHEMATICIAN

BRAHMAGUPTA

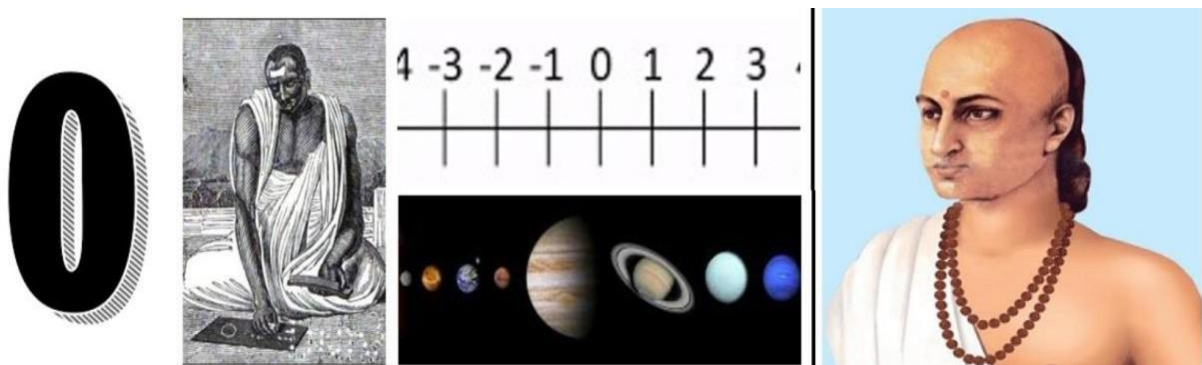
Renuka
M.Sc. 2nd Sem.



INTRODUCTION :-

Brahmagupta was an ancient Indian mathematician and astronomer who lived during the 6th and 7th centuries CE. He is considered one of the most significant figures in the history of Indian mathematics and made substantial contributions to various branches of mathematics and astronomy. Here are key aspects of Brahmagupta's life and work:

Brahmagupta was born in 598 CE in Bhinmal, a town in present-day Rajasthan, India. Not much is known about his personal life, but he likely lived during the Gupta dynasty, a period often referred to as the "Golden Age" of ancient Indian mathematics. Unfortunately, historical records about the personal life of Brahmagupta are limited, and not much is known about his personal details or experiences.



CONTRIBUTION:-

Brahmagupta made substantial contributions to arithmetic, including advancements in the understanding of zero and negative numbers. He provided rules for performing calculations with zero and negative numbers, although he considered the product of

two In the “*Brahmasphutasiddhanta*,” Brahmagupta presented solutions to quadratic equations, including both positive and negative roots. He introduced methods for solving linear and quadratic indeterminate equations.

Brahmagupta’s work in geometry included the calculation of the area of cyclic quadrilaterals. He provided a formula for the area of a cyclic quadrilateral in terms of its side lengths, known as Brahmagupta’s formula.

BRAHMAGUPTA’S WORK:-

- Brahmagupta’s most famous work is his *Brahmasphutasiddhanta*.
- Brahmagupta was the first to use zero as a number . He gave rules to compute with zero.
- Brahmagupta used negative number and zero for computing . The modern rule that two negative numbers multiplied together equals a positive number first appears in *Brahmasphutasiddhanta*.
- Brahmagupta gave the solution of the general linear equation.

BOOKS:-

Brahmagupta has written two famous books *Brahmasphutasiddhanta* and *Khanda Khadyaka* . The former has 24 chapters and contains 1008 verses consisting of mathematics and astronomy. In the first 10 chapters , he has discussed the matters that were known earlier to him and rest of the chapters deal with new topics discovered by him. The twelfth chapter of the *Brahmasphutasiddhanta* is devoted to arithmetic, geometry and mensuration in 56 verses . The 18th chapter consisting of 102 verses deals with algebra . Brahmagupta had written this book at an early of 30.

The men who invented zero

Roshan Kumar

M.Sc. 2nd Sem.



This blog focuses on the contribution of two Indian mathematicians to the concept of zero.

Aryabhata



Brahmaputra



What we now call zero in English, Brahmagupta named **shun'ya** or **sun'ya**, the Sanskrit word for emptiness or nothingness.

Aryabhata and Brahmagupta wrote their works in Sanskrit, an ancient and classical language of India. Their use of numbers would have looked quite different to what we use in The move from zero as merely a placeholder by the Mayans and Babylonians – a tool to distinguish larger numbers from smaller ones to a digit of its own was established in India by a man named Aryabhata in the 5th Century. A

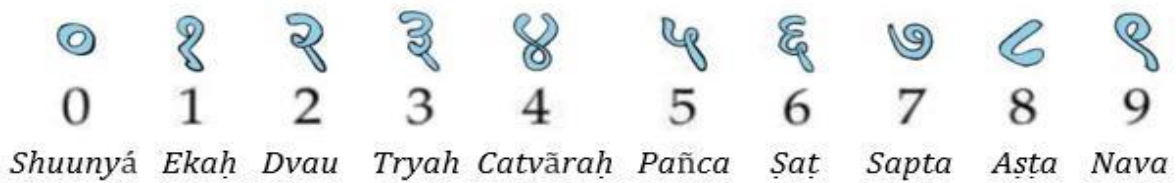
शून्य

/shūnya/

mathematician and astronomer, Aryabhata contributed multiple mathematical concepts, crucial to maths as we know it today, including the value of pi being 3.14 and the formula for a right-angled triangle. The prior absence of zero created difficulty in carrying out simple calculations.



English now. However, Sanskrit had a large Influence in how the English numeric system is written and so there are quite a lot of similarities. Following this in the 7th century a man known as Brahmagupta, developed the earliest known methods for using zero within calculations, treating it as a number for the first time. The use of zero was inscribed on the walls of the Chaturbhuj temple in Gwalior, India. Carved into a wall the numbers 270 and 50 can be seen today and have been established as the second oldest recorded zeros in history. The city of Gwalior was designed so that the gardens around the temple were large enough so that each day the gardens would produce enough flowers to create 50 garlands for the employees of the temple. When the temple was built this was inscribed on the walls and it is this 50 that can be seen, annotated almost as we would write it today.



Numbers 0 to 9 in Sanskrit

Within Indian culture there is an idea of one having a “nothing” or a void inside of yourself. Long before the conception of zero as a digit, this philosophical concept was taught within Hinduism and Buddhism and practised through meditation. The ancient Hindu symbol, the “Bindi” or “Bindu”, a circle with a dot in the centre symbolised this and was what probably led to the use of an oval as the symbol for the Sunya. It has been suggested that this cultural and philosophical influence on the concept of zero is what allowed India to develop what previous civilizations did not think of.

Brahmagupta was also the first to demonstrate that zero can be reached through calculation. He wrote these rules in his book the “Brahmasphutasiddhanta”. He was therefore able to make another important leap – in the creation of negative numbers, which he initially called “debts”. Brahmagupta placed small dots above numbers to indicate they were negative, unlike today where a minus symbol is used. The use of negative numbers was shown in “Brahmasphutasiddhanta”. Brahmagupta also demonstrated their use to produce the quadratic formula. And demonstrated rules for calculations involving both negative numbers and zero.

His rules were as follows:

Addition and Subtraction with zero and negative numbers:

- When zero is added to a number or subtracted from a number, the number remains unchanged; and a number multiplied by zero becomes zero.
- A debt minus zero is a debt.
- A fortune minus zero is a fortune.
- Zero minus zero is a zero.
- A debt subtracted from zero is a fortune.

- A fortune subtracted from zero is a debt.

Division and multiplication with zero and negative numbers:

- Positive or negative numbers when divided by zero is a fraction with zero as denominator.
- Zero divided by negative or positive numbers is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator.
- Zero divided by zero is zero.
- The product of zero and a debt or fortune is zero.
- The product of zero and zero is zero.
- The product or quotient of two fortunes is one fortune.
- The product or quotient of two debts is one fortune.
- The product or quotient of a debt and a fortune is a debt.
- The product or quotient of a fortune and a debt is a debt.

When Brahmagupta attempted to divide 0 by 0, he came to the result of 0. However, most modern mathematicians would argue that 0 divided by 0 is undefined, or an “indeterminate form”. Despite this outlier, the rest of Brahmagupta’s grasp on the number zero is exactly how we conceptualize it today.

The concept of zero gradually moved East into China. Then West to reach the Middle East. And finally, over half a century from its conception, the zero made it to Europe, where its importance was finally recognised by the Western culture that previously frowned on the idea of nothing, referring to it as meaningless or even satanic. In 1200 AD, Italian mathematician Fibonacci, a man who has been considered the ‘most talented western mathematician of the middle ages’ wrote of Indian Mathematics and their use of zero:

Despite the number zero having quite literally no value, its concept has allowed

"The method of the Indians surpasses any known method to compute. It's a marvelous method. They do their computations using nine figures and the symbol zero."

mathematics to develop into what it is today. Its curation led to the three pillars of modern mathematics: algebra, algorithms, and calculus. The use of calculus (the mathematical study of continuous change), which the zero is crucial for, has allowed engineering and modern technology to be possible. The use of zero and one within the binary system is what made computing possible. So, without the invention of zero much of what we know today would not have been possible. The device you are reading this on would not have been able to be invented, if not for Aryabhata, Brahmagupta and India’s fascination with the idea of nothing.

Applications of Trigonometry

INTRODUCTION

Trigonometry is the most effective branch of Mathematics, which includes tremendous number of applications in real life. The word trigonometry means calculations with triangles(that's where 'tri' comes from). Its study of relationships in Mathematics which involves lengths, heights and angles of different triangles. Trigonometry emerged during 3rd century B.C from applications of geometry to astronomical studies. Trigonometry spreads its applications into various fields such as architecture, engineering, astrophysics etc.



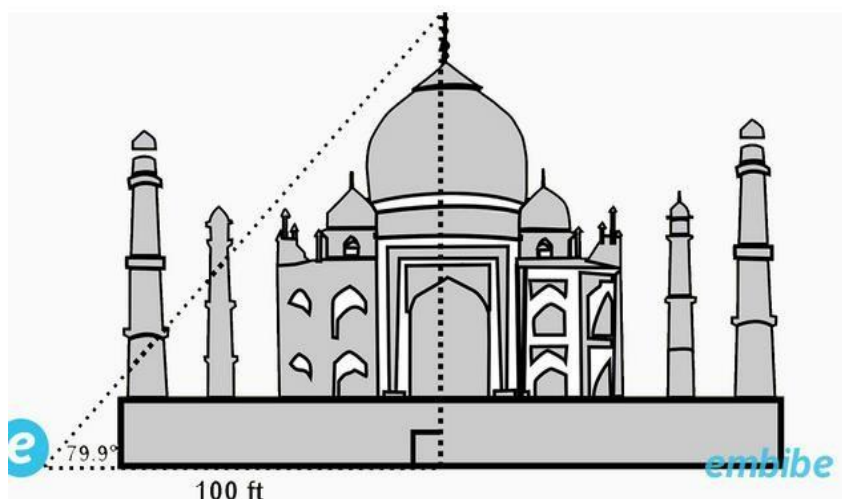
Ruchi Choudhary

M.Sc. 2nd Sem.

TRIGONOMETRY IN DAILY LIFE

In our daily life, there are lots of applications of trigonometry. It may not have direct applications in solving practical issues but it is used in various things that we enjoy very much. In this section, a few of basic and important applications are listed below;

1. MEASUREMENT OF HEIGHT OF A BUILDING OR MOUNTAINS



If an individual knows the distance between the observation point to the building and the angle of elevation, the height of a building can be easily found out. Even the height of the mountains was developed on the same principle.

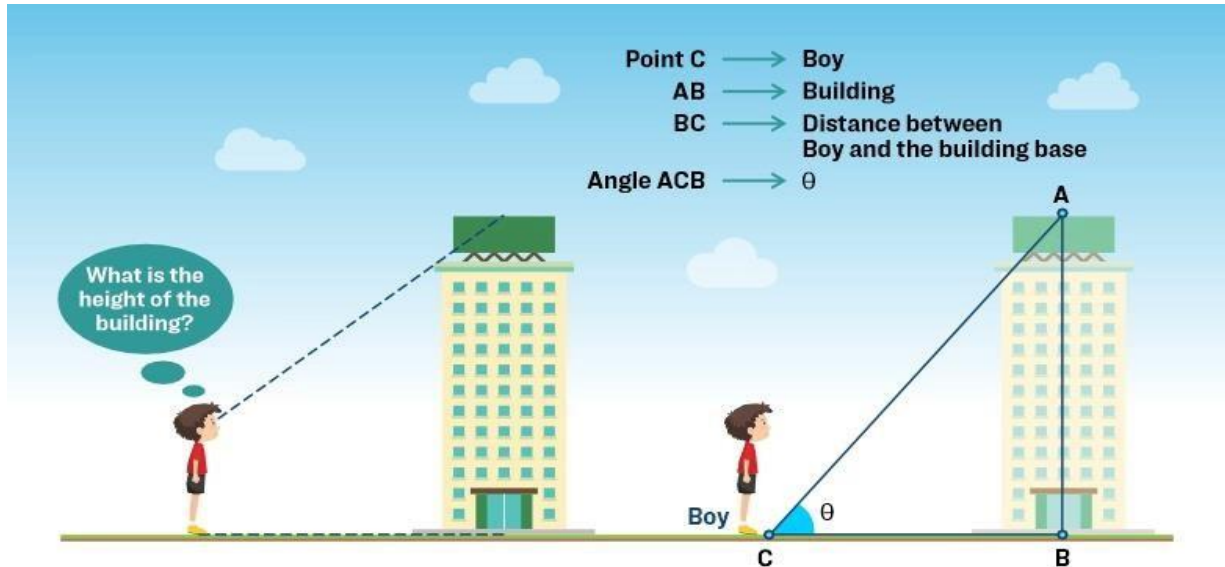
2. TRIGONOMETRY IN CONSTRUCTION

In constructions, we need Trigonometry to calculate the following

- Measuring fields, lots and areas
- Making walls parallel and perpendicular
- Installing ceramic tiles

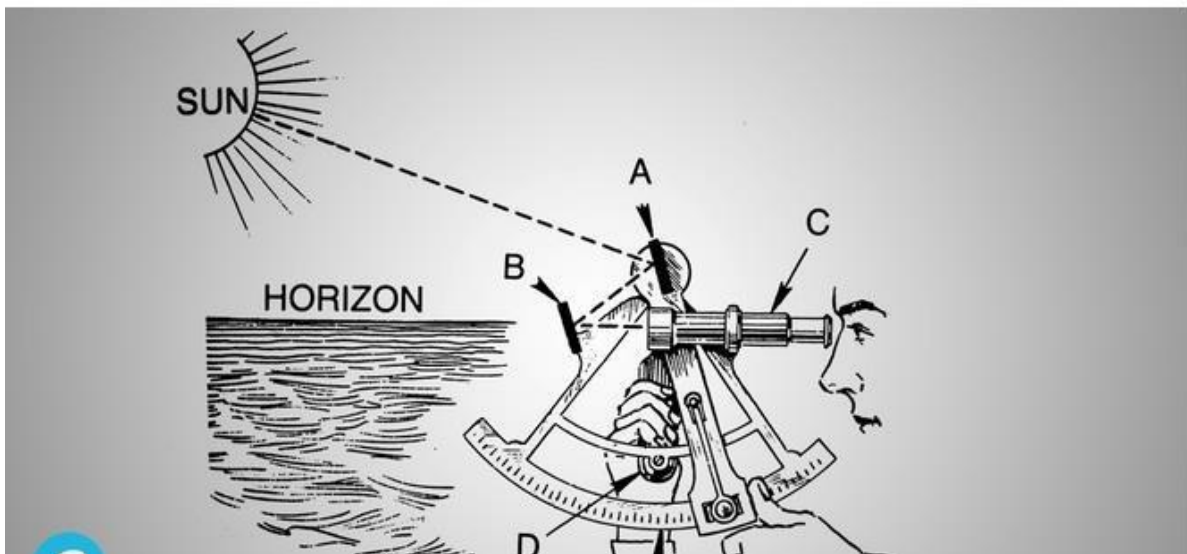
- roof inclination
- Architects use Trigonometry to calculate structural loads, roof slopes, ground surfaces and many other aspects including sun shading and light angles.

In all the above constructions trigonometry becomes essential branch of Mathematics.



3.Trigonometry used in navigation:

Trigonometry is used to set directions such as the north south east west, it tells you what direction to take with the compass to get on a straight direction. It is used in navigation in order to pinpoint a location. It is also used to find the distance of the shore from a point in the sea. It is also used to see the horizon.



4. TRIGONOMETRY IN MARINE ENGINEERING:

In marine engineering trigonometry is used to build and navigate marine vessels. To be more specific trigonometry is used to design the Marine ramp, which is a sloping surface to connect lower and higher level areas, it can be a slope or even a staircase depending on its application.



5. TRIGONOMETRY IN MARINE BIOLOGY:

Marine biologists often use trigonometry to establish measurements. For example, to find out how light levels at different depths affect the ability of algae to photosynthesize. Trigonometry is used in finding the distance between celestial bodies. Also, marine biologists utilize mathematical models to measure and understand sea animals and their behavior. Marine biologists may use trigonometry to determine the size of wild animals from a distance.

OTHER USES OF TRIGONOMETRY:

- ✓ It is used in oceanography in calculating the height of tides in oceans.
- ✓ The sine and cosine functions are fundamental to the theory of periodic functions, those that describe the sound and light waves.
- ✓ Calculus is made up of Trigonometry and Algebra.
- ✓ Trigonometry can be used to roof a house, to make the roof inclined (in the case of single individual bungalows) and the height of the roof in buildings etc.
- ✓ It is used naval and aviation industries.
- ✓ It is used in cartography (creation of maps).
- ✓ Also trigonometry has its applications in satellite systems

CONCLUSION

Thus there are tremendous applications of Trigonometry that rules the present world with its flexibility. The field of Trigonometry is an important branch of Applied Mathematics. A branch of Mathematics which was developed to solve certain issues of Physics is now an important field of Mathematics for its excellent applications in the modern world.

PRINCIPLE OF EQUIVALENCE

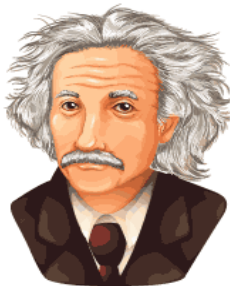
Sanajay Kumar

M.Sc. II Sem.



INTRODUCTION

According to principle of covariance the physical law must be expressed in the form of covariant tensor equation; so that they may remain unchanged in all coordinate system. But in order to introduce the effects of gravitational field in the relativistic theory of gravitation, Einstein gave another principle known as the principle of equivalence. Consider two systems s and s' , the latter moving with uniform acceleration relative to the former. If we consider the first frame s to be inertial, then second frame s' will be non-inertial and vice versa.

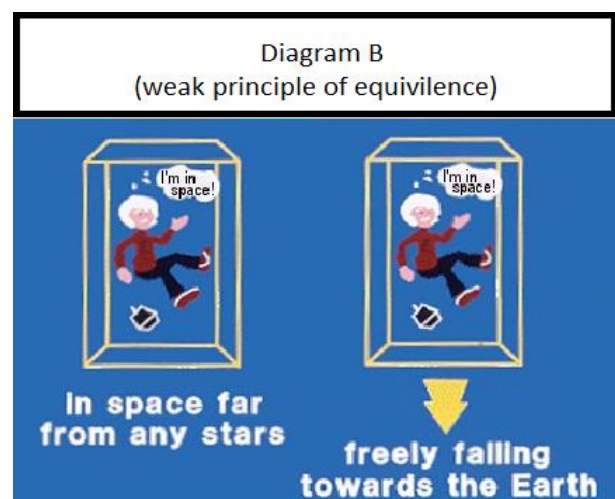


Einstein managed to fit a square peg into a round hole by modifying both the peg and the hole. His general theory of relativity resolved conflicts between Newton's theory of gravity and the special theory of relativity.

The principle of equivalence is a fundamental concept in Einstein's general theory of relativity. It states that the effects of gravity are locally indistinguishable from acceleration. There are two main forms of this principle

1. Weak Equivalence Principle (WEP):

Also known as the Galilean Equivalence Principle or Einstein's First Postulate, it states that the motion of a freely falling object in a gravitational field is independent of its mass or composition. This was tested by Galileo and later confirmed by experiments like those of Eötvös.



2. Strong Equivalence Principle (SEP):

Extends WEP by stating that the laws of physics (including gravity) are the same in a freely falling reference frame as they would be in the absence of gravity.

This means that an observer in a sealed, accelerating spaceship (without external reference points) would not be able to distinguish between acceleration and gravitational force.

This principle led Einstein to develop

General Relativity, where gravity is not a force but a curvature of spacetime caused by mass and energy.

Inertial Mass is the measure of the resistance provided by body opposing the action of an external force whereas gravitational mass is the coefficient which determines the attraction force by a body under a gravitational field.

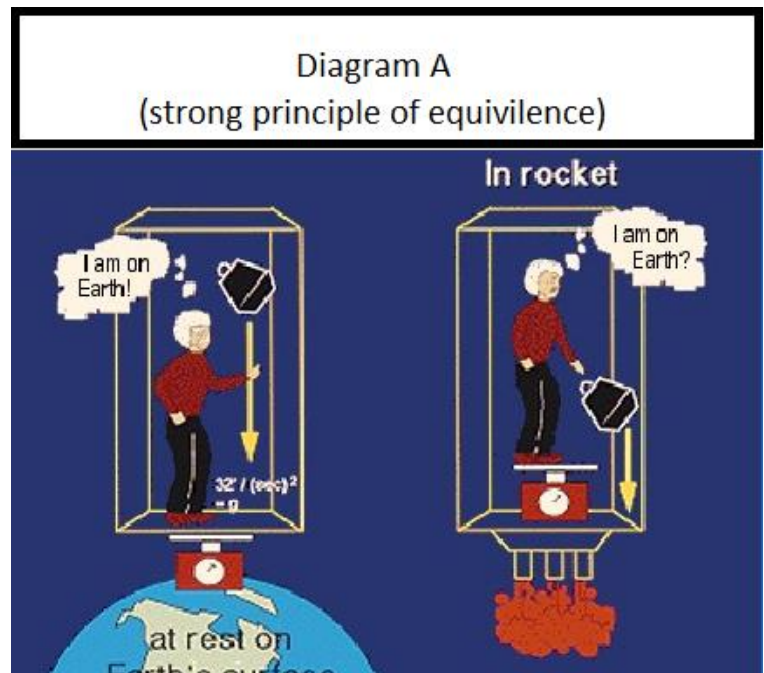
By this principle, if we apply the Newtonian theory then motion of a body under a gravitational field will be given by

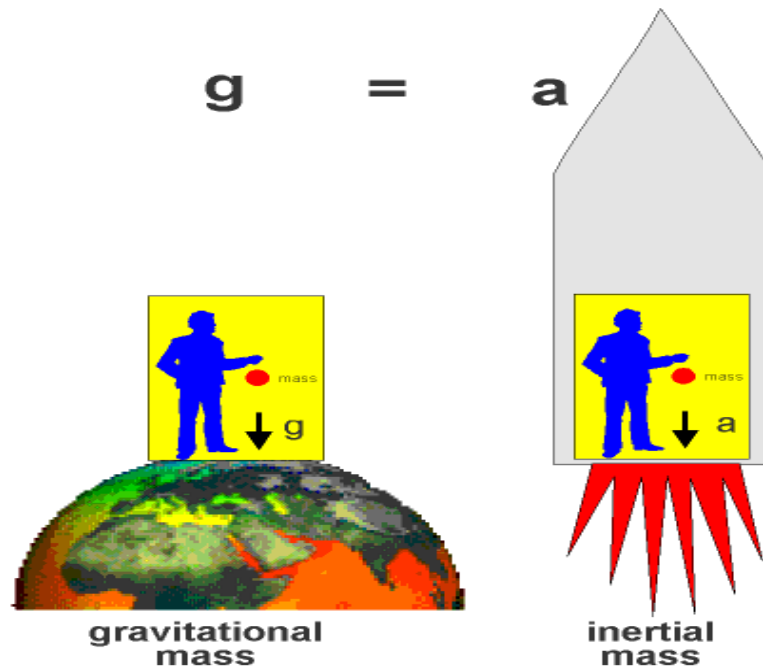
$$m_i \frac{\partial^2 x}{\partial t^2} = m_g \cdot g$$

$$m_i \frac{\partial^2 x}{\partial t^2} = \text{Inertial force}$$

$$\Rightarrow \frac{d^2 x}{dt^2} = +g$$

It was observed and experimentally verified by Galileo that many particles with different masses of all with the same acceleration under gravity which justified the above proof. With this principle we can say that the gravitational acceleration is similar to a relative acceleration which arises when a reference frame is subjected to accelerated motion to explain this Einstein gave a famous example of moving light.





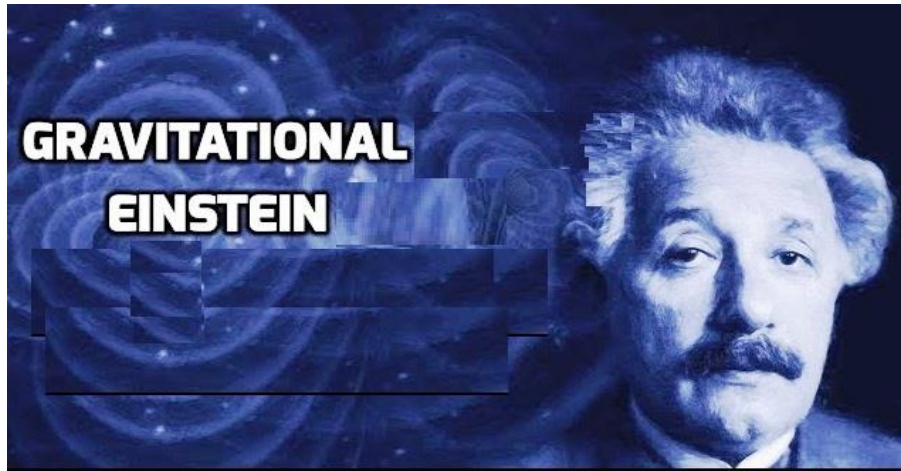
If a lift is going down with the acceleration equal to a gravitational acceleration g then the person on the lift will experience weightlessness. Conversely if lift moves upward then we get heavier. In this way we see that gravitational field can be produced by accelerating uniformly an inertial frame of ref.

The principle of General Co-variance-

The principle of general co-variance asserts that a physical equation is true in the presence of gravitation if

- (1) The equation is generally co-variant, i.e. it preserves its form under a general co-ordinate transformation $x \rightarrow x'$
 - (2) And the equation holds true in the absence of gravitation.
- In the principle of general co-variance we have remind that:
- i) Arbitrary co-ordinate transformation obviously include a subset of transformation that takes us (locally) from non-inertial to inertial frames. In the latter frames gravitational forces are absent. Now consider a generally covariant equation. If it is known to be true in locally inertial frames (i.e. in the absence of gravitation) then clearly it is true in every frame. In other words the equation is true in the presence of gravitation because gravitational forces are present in the non-inertial frames.
 - ii) The principle of general co-variance gives us a precise prescription through which we can introduce gravitational forces into a physical situation. If we know the equation describing the system in the absence of gravitation. For the latter equations we will take the laws of special theory of relativity (STR). Thus to write down equations valid in the corresponding equation of STR and then make them generally co-variant.

Finally, we came to the equation of a suitable mathematical apparatus through which the principle of general co-variance can be implemented. The apparatus is that of general tensors. (tensors defined with respect to arbitrary coordinate transformation). We now proceed to develop the formalism of general tensor



Einstein field equation of Gravitation-

The main principle of Einstein's theory of gravitation is that Gravitational field which are permanent in nature geometrizes the space-time present of gravitational field in the space make the space non-Minkowskian (curved space time). The Minkowskian space time is flat space. Presence of Gravitational field put curvature into it and the space time becomes curved.

Seeing the grand success of Newtonian theory for non-relative motion Einstein formulated the theory of gravitation in such a way that locally its equation must reduce to Newtonian results for gravitation.

In other words the proposed Einstein equation must generalize the well known Poisson's equation which describes gravitational field of continuum distribution of matter.

$$\nabla^2 \phi = -8\pi\rho$$

As we have seen in Newtonian approximation of equation of motion of a free particle under weak gravitational field that potential is related to main components of metric tensor of the space time.

The term $\nabla^2 \phi$ must be substituted by a tensor which is related to metric tensor as well as curved and R.H.S. $8\pi\rho$ must be related with a tensor which describes the matter distribution of space-time.

In fluid mechanics as well as in 'Electro dynamics' there is a popular tensor known as energy momentum tensor which describes the density as well as energy momentum density of the system. This is the symmetric tensor $T_{\mu\nu}$ (known as energy momentum).

Its divergence is zero $T_{0;j} = 0$

Einstein proposed this tensor as the replacement of R.H.S. $(-\nabla^2\phi)$ of the Poisson's equation in the generalized version of field equation of gravitation.

Likewise the left hand side term replacing $\nabla^2\phi$ in Poisson's equation must also be a tensor which is related with curvature (metric tensor) and which has same property of zero divergence like T_{ij} . Then inversion of Bianchi identity and then contracting it, in inversion of Einstein Tensor.

$$G^{ij} = R^{ij} - \frac{1}{2} R g^{ij}$$

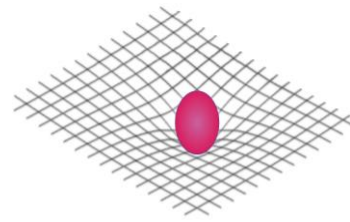
was formed to be suitable replacement of the left hand side terms of Poisson's equation. After lot of deliberation, the Einstein has put the following field equation for this gravitational field theory:

$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi G}{c^4} T_{ij}$$

Mathematical ecology employs two primary types of models: strategic and dynamical.

Strategic Models

1. **Empirical basis** : These models rely on empirical formulas.
2. **Simulation techniques** : They utilize computer simulation techniques.
3. **Predictive capabilities** : Strategic models are highly predictive for specific cases.
4. **Limitations** : They provide little insight into underlying ecological mechanisms.



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Dynamical Models

1. **Equations** : These models often employ ordinary differential equations (ODEs), but may also use stochastic differential equations, difference equations, integral equations, or diffusion reaction equations.
2. **Ecological mechanisms** : Dynamical models encode postulates about ecological mechanisms into the equations.
3. **Predictive capabilities** : These models generally predict less accurately than strategic models due to the constraints imposed by the postulates.
4. **Explanatory power** : Dynamical models provide tentative explanations for ecological phenomena, facilitating the development of improved strategic models for ecosystem management.

This summary focuses on ordinary differential equation models for brevity.

MATHEMATICAL ECOLOGY

Sanskar Tiwari
M.Sc. IVTH Sem.



Mathematical ecology employs two primary types of models: strategic and dynamical.

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Growth of a single population.

Let $N(t)$

denote the total number, or density, of a population Σ
at a fixed location and time. Assume that $N(t)$
is continuous in the time t

. The Hutchinson postulates [a16] are:

- 1) $dN/dt=f(N)$, f sufficiently differentiable;
- 2) $N \equiv 0$ implies $dN/dt \equiv 0$;

3) $N(t)$ is bounded between zero and a fixed positive constant C , for all time.

Given the Hutchinson postulates for a population Σ , it follows that the ordinary differential equation

$$\frac{dN}{dt} = \lambda N \left(1 - \frac{N}{K} \right), \quad (a1)$$

for which,

$$N(t) = \frac{K}{1 + be^{-\lambda t}} \quad (a2)$$

is the general solution, is the simplest growth law. It is called the logistic equation.

The parameter K , called the carrying capacity for Σ , obviously satisfies $0 < K \leq C$. The parameter $\lambda > 0$ is called the intrinsic growth rate. Of the four types of shapes specified for (a1) by

$b < 0$, $b = 0$, $0 < b \leq 1$, $b > 1$ only the last is S-shaped (i.e. its graph has an inflection point). Suppose that Σ satisfies only (a1) and (a2); then, denoting $n(t) = N(t) - N^*$, where, $f(N^*) = 0$ (i.e. N^* is a steady-state), Taylor expansion around N^* gives

$$\begin{aligned} \frac{dN}{dt} &= \frac{dn}{dt} = f(N) = \\ &= f(N^*) + f'(N^*)n + \frac{f''(N^*)}{2}n^2 + \dots, \end{aligned}$$

where the prime denotes differentiation with respect to N

. For $n(t)$

small in absolute value, dn/dt

is well approximated by $f'(N^*)n$

. Therefore, $n(t)$

increases with time if $f'(N^*) > 0$

, and decreases if $f'(N^*) < 0$

. In the former case, N^*

is an unstable steady-state while, in the latter case, N^*

is a stable steady-state.

For the logistic special case, $N^* = 0$

or $N^* = K$

are the only possible steady-states, the former being unstable and the latter stable.

The logistic differential equation (a1) is the simplest description of a population with limited resources, the limitation being provided by the negative coefficient of the quadratic term. The equation first arose in the work of P. Verhulst (1838) and later in

the demographic research of R. Pearl and L. Reed in the 1920s. It was subsequently used to provide a dynamic model of malaria in humans by Sir Ronald Ross, but has perhaps a more basic role in ecology than in epidemiology.

Growth dynamics in a competitive community.

Several species living in the same locality must forage for food and seek nesting sites in a field or stream, etc. These populations may or may not affect one another.

Suppose that n

species comprise a community Σ

in which there are no inter-specific interactions. This ecosystem can be modeled by

$$\frac{dN^i}{dt} = \lambda_{(i)} N^i \left(1 - \frac{N^i}{K_{(i)}} \right), \quad i = 1, \dots, n, \quad (\text{a3})$$

where N_i

denotes the total number or density of the i th species in Σ . This system has $2n$

steady-states, but only $(K(1), \dots, K(n))$

is stable. The equations (a3) describe non-competition.

Now suppose there is competition for food items, etc. How does one describe this? G.F.

Gause and A.A. Witt answered this for a 2

-species community ($n=2$) with

$$\begin{cases} \frac{dN^1}{dt} = \lambda_{(1)} N^1 \left(1 - \frac{N^1}{K_{(1)}} - \delta_{(1)} \frac{N^2}{K_{(1)}} \right), \\ \frac{dN^2}{dt} = \lambda_{(2)} N^2 \left(1 - \frac{N^2}{K_{(2)}} - \delta_{(2)} \frac{N^1}{K_{(2)}} \right). \end{cases} \quad (\text{a4})$$

Here, all λ , K and δ

are positive. This system has exactly one positive equilibrium $(N1^*, N2^*)$, given by

$$\begin{cases} N_*^1 = \frac{K_{(1)} - \delta_{(1)} K_{(2)}}{1 - \delta_{(1)} \delta_{(2)}}, \\ N_*^2 = \frac{K_{(2)} - \delta_{(2)} K_{(1)}}{1 - \delta_{(1)} \delta_{(2)}}. \end{cases} \quad (\text{a5})$$

If both numerators and denominators are positive, then $(N1^*, N2^*)$

in (a5) is stable. If they are both negative, (a5) is unstable. This is easily proved by using the stability Ansatz: the eigenvalues of the Jacobian of the right-hand side of a system

$$\frac{dN^i}{dt} = f^i(N^1, \dots, N^n), \quad i = 1, \dots, n,$$

evaluated at a steady-state $(N1^*, \dots, Nn^*)$

, must have negative real part for stability. If any of these is positive, an unstable case results.

In the question of survival for the two populations in Gause–Witt competition (a4), (a5), there are four cases to consider:

A) If $\delta(1) > K(1)/K(2)$ and $\delta(2) > K(2)/K(1)$, then (a5) is unstable, with survival depending on the initial proportions of N_1 and N_2 .

B) If $\delta(1) > K(1)/K(2)$ and $\delta(2) < K(2)/K(1)$, then (a5) is unstable, and the first species will be eliminated.

C) If $\delta(1) < K(1)/K(2)$ and $\delta(2) > K(2)/K(1)$, then (a5) is unstable, and the second species will be eliminated.

D) If $\delta(1) < K(1)/K(2)$ and $\delta(2) < K(2)/K(1)$, then (a5) is stable.

Therefore, only in case D), called incomplete competition, can both species coexist. This case translates as some geometrical separation of the two species, where the more vulnerable one has a refuge it can retreat to, or some resource available that the otherwise better adapted competitor cannot use.

Experiments performed by Gause on *Paramecium* verified the outcomes A)–D) qualitatively. Thus, the Gause–Witt equations imply that complete competitors cannot coexist. This is the famous principle of competitive exclusion, a corner-stone of mathematical ecology. There are variants and generalizations of this principle; see, e.g., [a2]. This generality underscores the fundamental importance of that principle. Indeed, biologists claim that competition between species has profound evolutionary consequences.

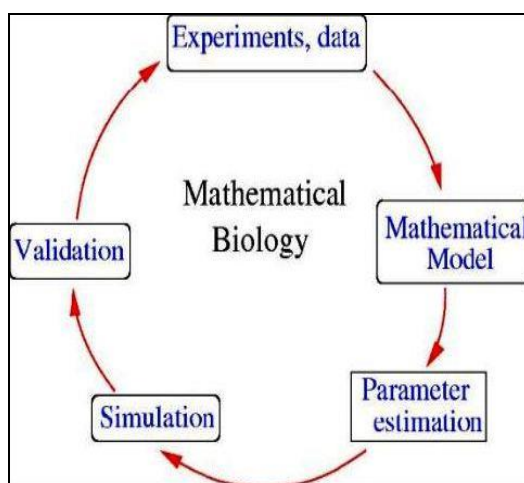
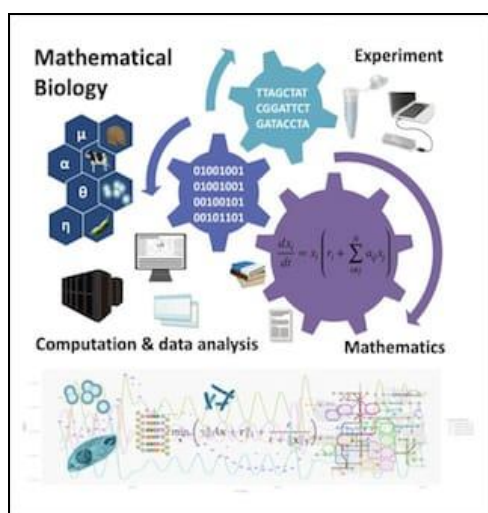
APPLICATION OF BIOLOGICAL MATHEMATICS IN MEDICAL SCIENCE

Seema Baghel
M.Sc. IVTH Sem.



INTRODUCTION

Mathematics for biosciences helps us to understand many biological and medical phenomena from those topics such as population growth, biology oscillations, pattern formation, and the spread of disease, human physiology, systems and organs, the development of tumour's, etc. It also produces new mathematical questions. The use of mathematics in biology for the raw data mass is in tracking change over time. Bio-statistics uses statistical analyses to form biological phenomena such as drawing balancing or connections between biological-Variables. One of the application of mathematics in biomedical is the use of probability and statistics, invalidating the effectiveness of drugs. Along with this mathematical field in medicine is to evaluate the survival rate of cancer patients under the treatments.



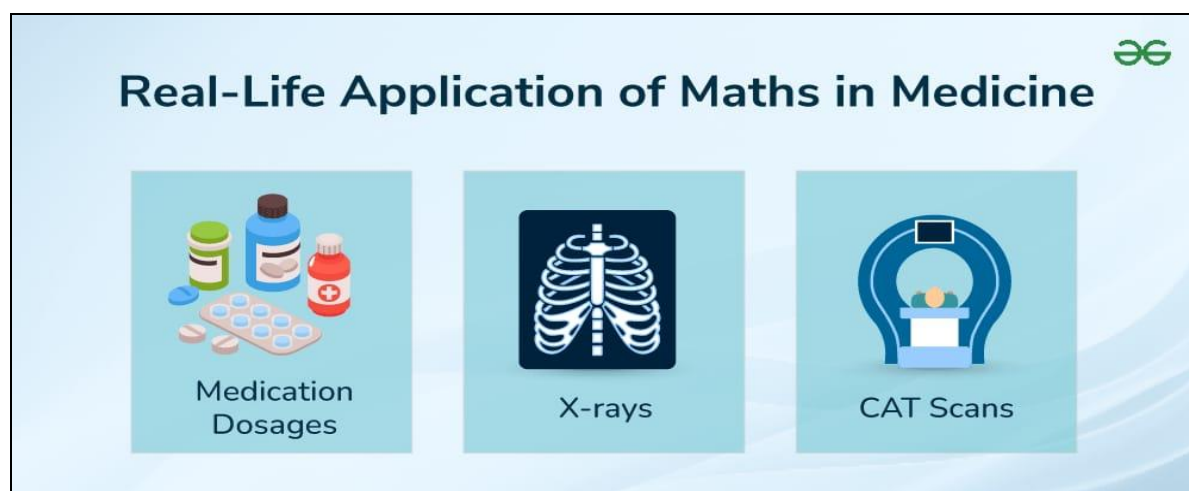
METHODOLOGY

Calculus is a branch of mathematics that is applied to real-life problems. Calculus is a mathematical study of continuous change, and it can be broken down into two types: Differential and Integral calculus. calculus can be used in fields such as engineering, economics, and medicines.

The purpose of this presentation is to explain the real-life application of calculus, especially in the medical field. Calculus has many vital applications in medicine when considering drug dosages, blood flow, and tumour growth. This is important because calculus can be used not to

solve and analyse real-life problems, but also to make medical procedures more efficient and beneficial to the public.

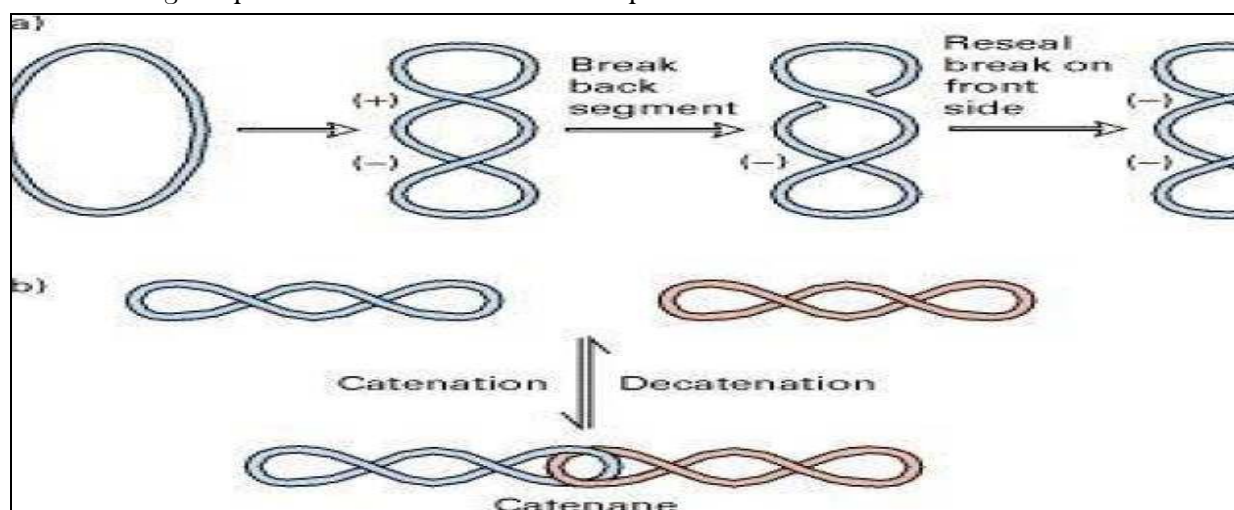
Calculus is often used in medicine to determine the right dosage of a drug to be administered to a patient. This is very important when taking into consideration drug sensitivity, rate of dissolution, and blood pressure.



CALCULUS CAN BE APPLIED TO DRUG SENSITIVITY

The concept of derivatives can be applied to determine the patient's sensitivity to a specific drug, to prevent too much of that drug from being administered. For example, if a drug's potency is given by $P(t)$ where t is a dosage, the derivative $P'(t)$ gives the sensitivity of the body with respect to t . $P'(t)$ also represents the change in P as a result of the change in the dosage of the drug.

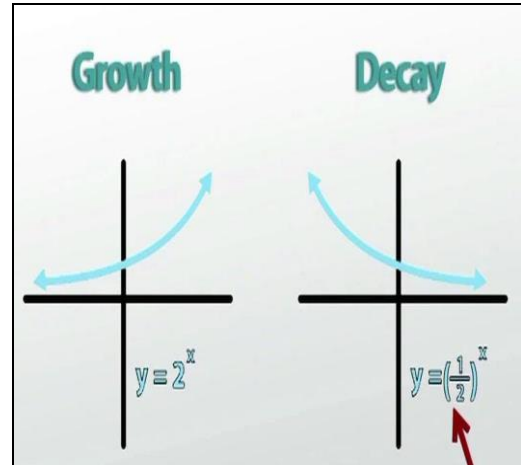
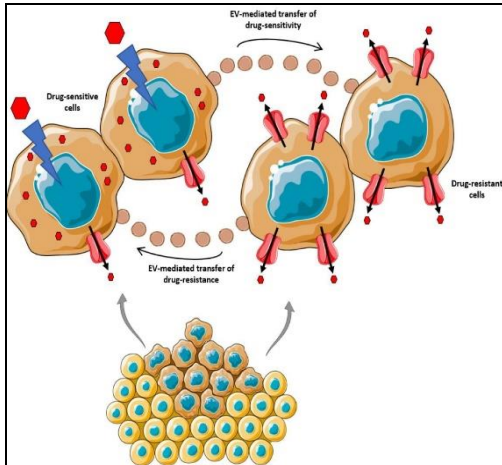
Another topic covered in calculus that can be applied to medicines is optimization. Doctors have to prescribe the right dosage of medicines to not only produce the desired effect but also to minimize negative side effects. In order to do this, optimization is used to determine the correct dosage to produce the best result in each patient.



Exponential Growth/Decay

This calculus topic can be used in the medical field to analyze the growth of bacteria in a patient, so that doctors may combat the bacteria with antibiotics.

A patient has been exposed to bacteria in her workplace. Her lab results showed 350 bacteria in her bloodstream at time $t=2$ hours and 780 bacteria at time $t=4$ hours. Find the rate at which the bacteria multiplied. How many bacteria were present initially? Set up an equation that shows the exponential growth of this bacteria and use it to find how many bacteria were present at time $t=24$ hours.



Exponential Growth Formula: $y = Ce^{kt}$

C = Initial Value

K = Rate of growth/decay

T = Time

RATE OF EXPONENTIAL GROWTH

$$350 = Ce^{2k}, 780 = Ce^{4k}$$

$$C = 350/e^{2k}, C = 780/e^{4k}$$

$$350e^{4k} = 780e^{2k}$$

$$e^{4k} = 2.229e^{2k}$$

$$\ln e^{4k} = \ln 2.229e^{2k}$$

$$4k = \ln 2.229 + 2k$$

$$2k = \ln 2.229$$

$$K = 0.4008.$$

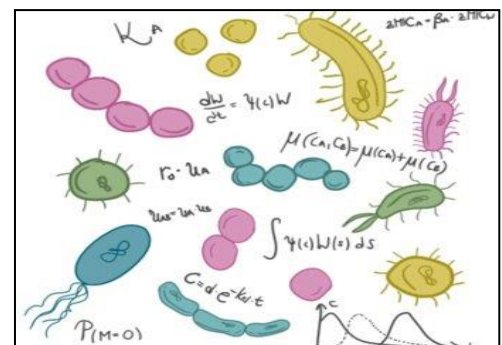
The rate of exponential growth is 0.4008 bacteria per hour.

BACTERIA PRESENT INITIALLY

$$C = 350/e^{2(0.4008)}$$

$$C = 157.021$$

There were 157.021 bacteria present initially.

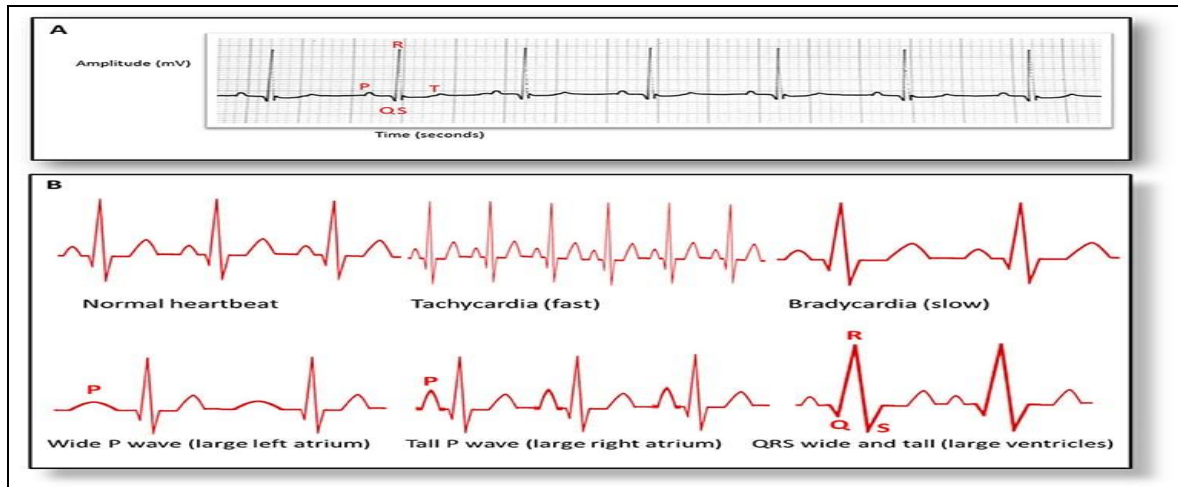


EXPONENTIAL GROWTH EQUATION

$Y = 157.021e^{0.4008t}$ ----- Equation of exponential growth

$Y = 157.021e^{0.4008(24)}$

$Y = 2,363,323.749$ ----- There is 2,363,323.749 bacteria present at time $t = 24$ hours.



CONCLUSION

Calculus has many real-world applications in a variety of fields. Calculus also holds significant importance in the medical field, especially when analysing problems. The use of calculus is to solve a problem and make medical procedures more efficient.

BOUDHAYANA: THE INDIAN MATHEMATICIAN & VEDIC SAGE



Shilpi Nishad
M.Sc. 2nd Sem.



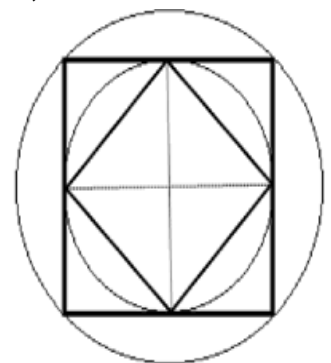
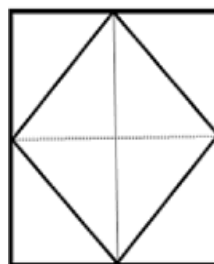
INTRODUCTION

Geometry of Euclid is taught all over the world, considering it to be authentic in the subject of geometry. But it should be remembered that even before the great Greek geologist Euclid, many geometry scientists in India had discovered important laws of geometry, among those geometry the name of Boudhayana is paramount. Geometry or Geometry in India at that time was called Shulba Sutras. The sutras of Baudhayana are in Vedic Sanskrit and are related to religion, daily rituals, mathematics etc. They related to the Taittiriya branch of the Krishna Yajurved. These are probably the oldest texts in the Sutra texts. They were probably composed in the 8th-7th century BC. Most notably, Boudhayana's Shulbasutras contain many results and theorems of early mathematics and

geometry including an approximate value of the square root of 2, and a statement of the Pythagorean theorem

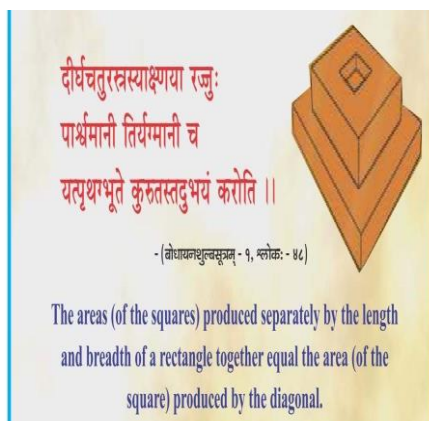
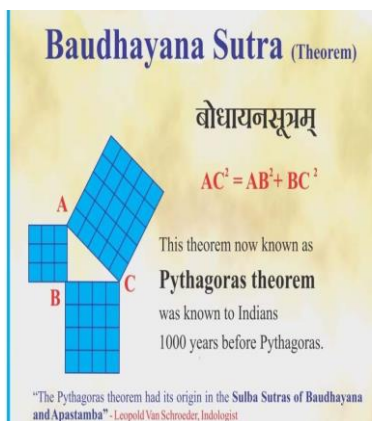
WORK OF BODHAYANA:

Boudhayana verse number i.61-2 (explained in Apastamba i.6) describes the method of finding the length of a diagonal given the lengths of the sides of a square. In other words, it describes



the method of finding the square root of 2. The verse related to this solution is as follows:-

To get the value of the diagonal of a square, by adding one-third to the side, then adding one-fourth of it, then subtracting thirty-fourth of it, what is



He knew that the area of a circle is proportional to the square of its radius and the above construction proves to be the same. By the

obtained is approximately the value of the diagonal.

Draw half its diagonal about the center towards the East-West line; then describe a circle together with a third part of that which lies outside the square. That is, if the side of the square is $2a$, then the radius of the circle $r = [a + 1/3(\sqrt{2}a - a)] = [1 + 1/3(\sqrt{2} - 1)] a$ if you wish to turn a circle into a square, divide the diameter into eight parts and one of these parts into twenty-nine parts: of these twenty-nine parts remove twenty-eight and moreover the sixth part (of the one part left) less the eighth part (of the sixth part).

Boudhayana was able to draw a circle roughly equal to the area of a square and vice versa. These processes are described in their sources (I-58 and I-59). Possibly in his quest to build circular altars, he constructed two circles enclosing the two squares shown below. Now, as in the area of squares, he realized that the inner circle should be exactly half of the larger circle in area.

same logic, as the circumference of two squares, the circumference of the outer circle must also be 22 times the circumference of the inner circle. This proves the known fact that the perimeter of a circle is proportional to its radius. This led to an important perusal made by Boudhayana. that the areas and perimeters of many regular polygons, including the squares above, can be related to each other as in the case of circles

1. na theorem

Boudhayana listed the Pythagorean theorem in his book Boudhayana Sulbasutra.

For example Boudhayana used a rope in the above verse/verse, which can be translated as: The areas produced by the length and breadth of a rectangle separately are equal to the areas produced by the diagonal together. The described diagonals and sides are those of a rectangle, and the areas are those of the squares whose sides are those of these line segments. Since the diagonal

of a rectangle is the hypotenuse of a right triangle formed by two adjacent sides, this statement appears to be equivalent to the Pythagorean theorem. Various arguments and explanations have been given for this. While some have assumed that the sides relate to the sides of a rectangle, others say that the reference may be to a square. There is no proof to suggest that Baudhayana's formula is limited to right-angled isosceles triangles so that it can be linked to other geometric figures as well. So it is logical to assume that the sides he mentioned can be sides of a rectangle.

Boudhayana seems to have simplified the learning process by encapsulating the mathematical result in a simple verse in a layman's language. As we see, it becomes clear that this is probably the most innovative way to understand and visualize the Pythagorean theorem (and geometry in general). Comparing his findings with Pythagoras' theorem: In mathematics, the Pythagorean theorem is a relation between the three sides of a right-angled triangle. According to this theorem area of square at hypotenuse is equal to sum of areas of square at other two sides. If c is the length hypotenuse of the right angled triangle with a and b being the other two then The question

may well be asked why the theorem is attributed to Pythagoras and not Boudhayana. Boudhayana used area calculations and not geometry to prove his calculations. He came up with geometric proof using isosceles triangles.

2.Value of π

Boudhayana is considered among one of the first to discover the value of 'pi'. There is a mention of this in his Sulbha sutras. According to his premise, the approximate value of pi is 3.3. Several values of π occur in Bodhayana's Sulbasutra, since, when giving different constructions, Boudhayana used different approximations for constructing circular shapes. Some of the major theorems propounded by Boudhayana are:

1. Diagonals of a rectangle cut each other at their middle point.
2. Diagonals of a rhombus cut each other at ninety degree
3. The area of a square formed by joining the mid-points of the sides of a square is half the area of the original square.
4. A rhombus is formed by joining the mid-points of the sides of a rectangle whose area is half the area of the original rectangle

MATHEMATICS IN HUMAN BODY

Shriya Rawate
M.Sc. 2nd Sem.



INTRODUCTION-

Mathematics plays a crucial role in understanding and analyzing the human body, from calculating medication dosages and monitoring vital signs to modeling biological processes and interpreting medical data.

Here's a more detailed look at how math is applied in the context of the human body:

- **Anatomy and Physiology:**

1. **Body Mass Index (BMI):**

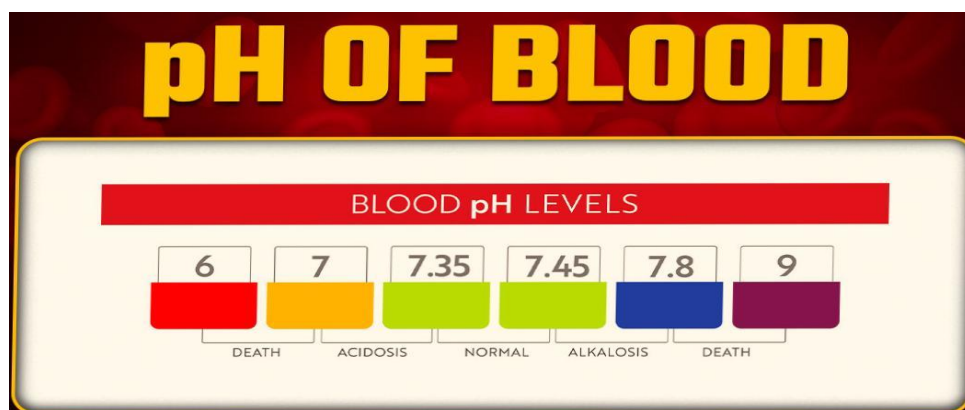
BMI, a common measure of body fat, uses height and weight measurements to determine if someone is at a healthy weight.

2. **Blood pH:**

Calculating the negative logarithm of the hydrogen ion concentration determine blood pH, a crucial indicator of bodily health.

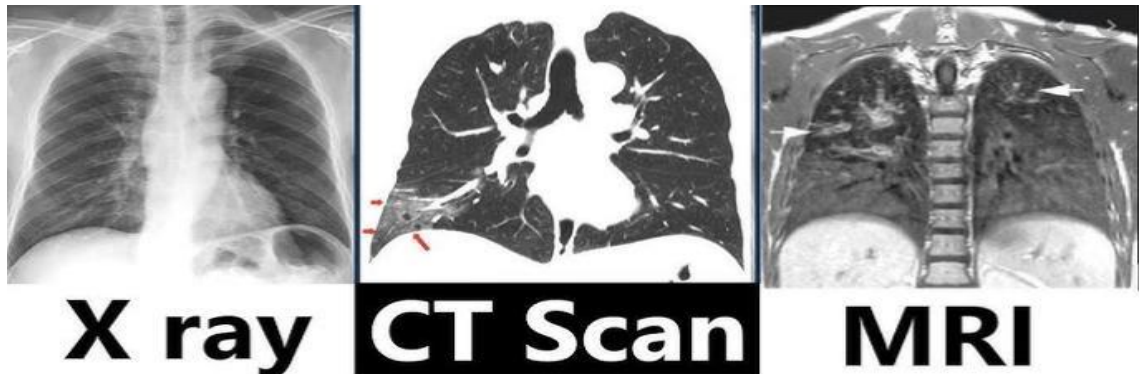


$$\text{BMI} = \frac{\text{Weight in kilogram}}{(\text{Height in meter})^2}$$



3. Medical Imaging:

Techniques like X-rays, CT scans, and MRI rely on mathematical principles for accurate image formation and interpretation.



4. Mathematical Models:

Mathematical models are used to simulate biological processes, disease progression, and treatment effects, aiding in predicting outcomes and optimizing treatments.

5. Allometric Scaling:

Understanding how metabolism and other physiological processes scale with body size (allometric scaling) is a fascinating area of mathematical application in biology.

• Healthcare Applications:

1. Medication Dosage:

Accurate calculations are essential for determining appropriate drug dosages, especially considering factors like body weight and age.

Basic dosage calculations	
$\frac{D \text{ (desired dose)}}{H \text{ (amount on hand)}} \times V \text{ (volume)} = \text{Dose}$	
I.V. drips in mcg/minute	
$\frac{\text{mg}}{\text{mL}} \times \frac{1,000 \text{ mcg}}{1 \text{ mL}} \times \frac{\text{mL}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ minute}} = \text{mcg/minute}$ (÷ by kg to get mcg/kg/minute)	
I.V. drips in unit per hour	
$\frac{D \text{ (desire)}}{H \text{ (on hand)}} \times V \text{ (volume)} = \text{units/hour (\# mL} \times \text{units/mL} = \text{dose)}$	
Dosage calculation conversions	
1 mg = 1000 mcg	1gm = 1000 mg
1 L = 1000 mL	1 mL = 1 cc
5 mL = 1 Tsp	3 Tsp = 1 Tbsp
15 mL = 1 Tbsp	30 mL = 1 oz
1 oz = 2 Tbsp	8 oz = 1 Cup
1 kg = 1000 gm (g)	1 kg = 2.2 lbs

2. Monitoring Vital Signs:

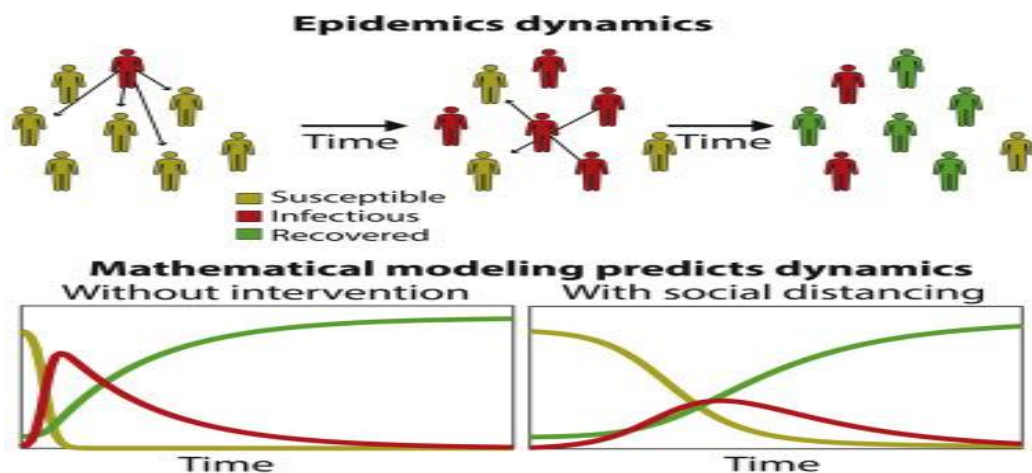
Tracking pulse rate, breathing rate, blood pressure, and body temperature relies on mathematical measurements and analysis.

Blood pressure chart by age			
Age	Min	Normal	Max
1 to 12 months	75/50	90/60	110/75
1 to 5 years	80/55	95/65	110/79
6 to 13 years	90/60	105/70	115/80
14 to 19 years	105/73	117/77	120/81
20 to 24 years	108/75	120/79	132/83
25 to 29 years	109/76	121/80	133/84
30 to 34 years	110/77	122/81	134/85
35 to 39 years	111/78	123/82	135/86
40 to 44 years	112/79	125/83	137/87
45 to 49 years	115/80	127/84	139/88
50 to 54 years	116/81	129/85	142/89
55 to 59 years	118/82	131/86	144/90
60 to 64 years	121/83	134/87	147/91

• Data Analysis and Statistics:

1. Epidemiology:

Mathematical models help track and predict the spread of diseases, enabling public health interventions.



2. Clinical Trials:

Statistical analysis is used to determine the effectiveness of new treatments and drugs.

3. Medical Research:

Researchers use mathematical models to understand complex biological processes and develop new therapies.

4. Data Visualization:

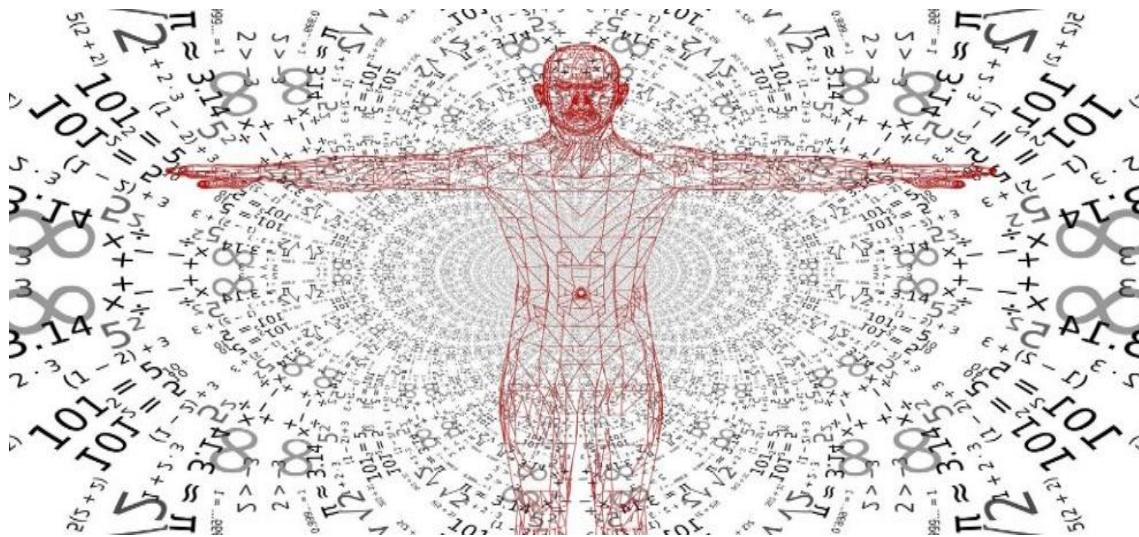
Graphs and charts are used to present medical data in a clear and understandable way.

- **Basic Mathematics used in human body:**

- 1) Arithmetic- Used for calculations like measuring body temperature, blood pressure, and calculating dosages of medication.
- 2) Algebra- Used in medical research and interpreting data, as well as in calculating body mass index (BMI).
- 3) Geometry- Important for understanding the shapes and structures of the body, including bones, organs, and their spatial relationships.
- 4) Trigonometry- Used in understanding the movements of joints and limbs, as well as in medical imaging.

- **Advanced Mathematics used in human body:**

- 1) Calculus-Used in modeling physiological processes, such as fluid dynamics in the circulatory system and the rate of drug absorption.
- 2) Statistics- Used to analyze medical data, interpret research findings, and understand disease patterns.
- 3) Probability- Used in understanding the likelihood of certain events, such as the risk of developing a disease or the effectiveness of a treatment.



Inverse tan is the inverse function of the trigonometric function ‘tangent’. It is used to calculate the angle by applying the tangent ratio of the angle, which is the opposite side divided by the adjacent side of the right triangle. Based on this function, the value of $\tan^{-1} 1$ or $\arctan 1$ or $\tan^{-1} 10$, etc. can be determined. It is naming convention for all inverse trigonometric functions to use the prefix ‘arc’ and hence inverse tangent is denoted by \arctan . Although it is not uncommon to use \tan^{-1} , we will use \arctan throughout this article.

Inverse Tangent Formula

Addition Formula:

The formula for adding two inverse tangent function is derived from \tan addition formula.

$$\tan a \pm b = \frac{\tan a \pm \tan b}{1 \mp \tan(a) \tan(b)}$$

In this formula, by putting $a = \arctan x$ and $b = \arctan y$, we get

$$\arctan(x) \pm \arctan(y) = \arctan\left(\frac{x \pm y}{1 \mp xy}\right) \pmod{\pi}, xy \neq 1$$

For Integration:

Some of the important formulae for calculating integrals of expressions involving the \arctan function are:

$$\int \arctan(x) dx = \{x \arctan(x)\} - \{\ln(x^2+1)/2\} + C$$

$$\int \arctan(ax) dx = \{x \arctan(ax)\} - \{\ln(a^2x^2+1)/2a\} + C$$

$$\int x \arctan(ax) dx = \{x^2 \arctan(ax)/2\} - \{\arctan(ax)/2a^2\} - \{x/2a\} + C$$

$$\int x^2 \arctan(ax) dx = \{x^3 \arctan(ax)/3\} - \{\ln(a^2x^2+1)/6a^3\} - \{x^2/6a\} + C$$

Calculus of Arctan Function

This section gives the formula to calculate the derivative and integral of the \arctan function.

Derivative of tan inverse x

The differentiation of $\tan^{-1} x$ is given below. The derivative of $\arctan x$ is denoted by $d/dx(\arctan(x))$ and for complex values of x , the derivative is equal to $1/(1+x^2)$ for $x \neq -i, +i$. Therefore, the differentiation of $\tan^{-1} x$ is given by:

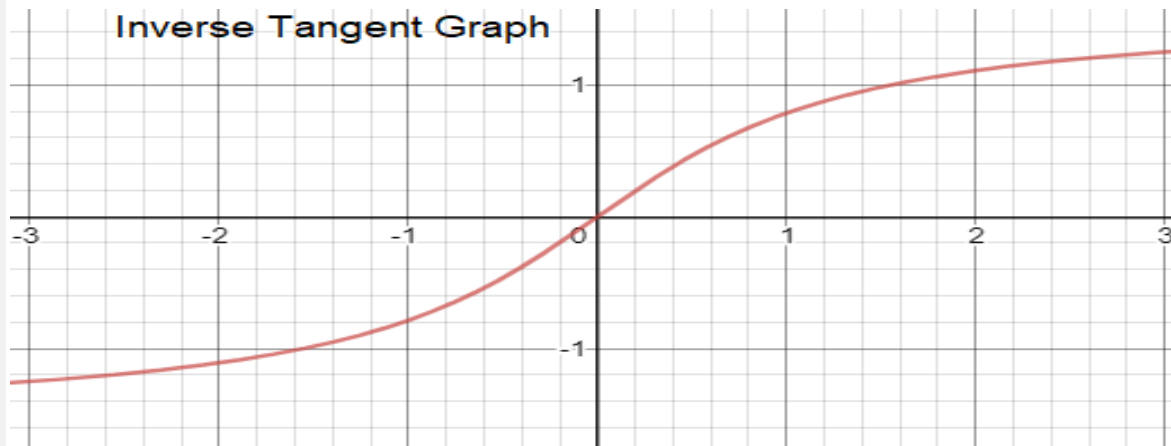
$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

Integral of inverse tan

For obtaining an expression for the definite integral of the inverse tan function, the derivative is integrated and the value at one point is fixed. The expression is:

$$\arctan(x) = \int_0^x \frac{1}{1+y^2} dy$$

Inverse Tan Graph



Relationship Between Inverse Tangent Function and Other Trigonometric Functions

Consider a triangle whose length of adjacent and opposite are 1 and x respectively. Therefore, the length of the hypotenuse is $\sqrt{1+x^2}$. For this triangle, if the angle θ is the arctan, then the following relationships hold true for the three basic trigonometric functions:

$$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$$

$$\cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$$

$$\tan(\arctan(x)) = x$$

Inverse Tangent Properties

The basic properties of the inverse tan function (\arctan) are listed below:

Notation: $y = \arctan(x)$

Defined as: $x = \tan(y)$

Domain of the ratio: all real numbers

Range of the principal value in radians: $-\pi/2 < y < \pi/2$

Range of the principal value in degrees: $-90^\circ < y < 90^\circ$

What is the Value of \tan^{-1} Infinity?

To calculate the value of the tan inverse of infinity (∞), we have to check the trigonometry table. From the table we know, the tangent of angle $\pi/2$ or 90° is equal to infinity, i.e.,

$$\tan 90^\circ = \infty \text{ or } \tan \pi/2 = \infty$$

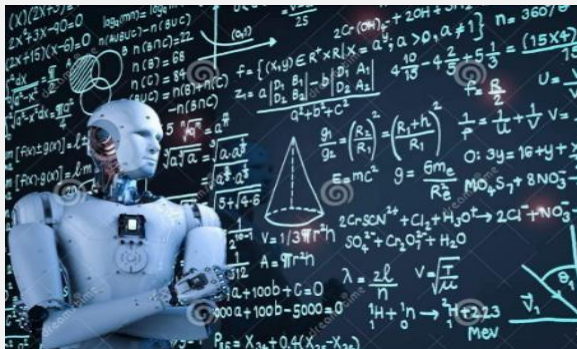
$$\text{Therefore, } \tan^{-1}(\infty) = \pi/2 \text{ or } \tan^{-1}(\infty) = 90^\circ$$

Application of mathematics In robotics

Mathematics is indispensable in robotics, serving as the foundation for designing, controlling, and optimizing robotic systems, from basic arithmetic to advanced calculus and statistics. It enables robots to perform complex tasks, understand their environment, and interact with humans effectively.



Venu Verma
M.Sc. 2nd Sem.



Robotics

summarizes the aspects of designing, building and analyzing robots. It poses a large number of challenges that deals with theoretical and vast mathematics like highly nonlinear, often algebraic problems in a moderate number of variables. Due to the nonlinearities the relevant (systems of) equations usually have multiple solutions. Further, safety considerations or the demand for a complete analyses require a worst case analysis of the possible scenarios.

Math helps in Robotics with many of its concepts having applications in learning robotics. A robot is a machine made by human beings using science, engineering, and technology. It replicates human actions and assists them. :

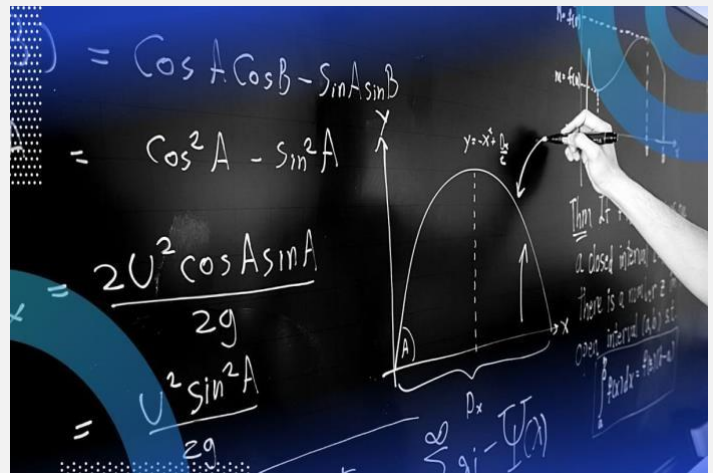
Contribution

- Algebra: It is that part of math that deals with symbols to represent numbers and quantities to deal with equations. Algebra has patterns and relationships which are used in the concepts of speed, time, distance, and force in Robotics.
- Making Robotics requires the use of Geometry. Geometry is part of Mathematics that studies the sizes, shapes, positions angles, and dimensions. Many robots are made with the use of wheels requiring the concepts of circles, area, and perimeter.

Concepts of Statistics is also used in Robotics. The robots once made are required to perform certain tasks. Data is analyzed for the Robots for their speed, use of force, the performance of tasks

The Benefits of Mathematics in Robotics Programming

Math is an invaluable tool for robotic programming, especially when it comes to manufacturing. Applications of mathematics in robotics are everywhere – from understanding how robots interact with their environment to making sure components of a factory move precisely from one point to another. Every part of every interaction must be mapped out, somewhere, by someone, even with the advent of newer programming methods that make the user experience a bit more friendly. Should that system fail, or should it be unavailable, you'll need to understand the math behind the programming of those actions.



With the use of math, robotics can be programmed in more sophisticated and efficient ways as equations can be used as a standard for complex calculations relating to robots' motion, force and speed. Additionally, math can provide a common language for all parties involved in robotic programming, unifying the efforts of engineers and coders from different backgrounds in order to create new technologies that will improve factory productivity, allowing modern robotics and manufacturing facilities to become even more efficient.

Importance of Mathematics in Robotics

Precision and Accuracy : Mathematics allows for precise control and manipulation of robots, ensuring accurate performance.

Efficiency and Reliability: Mathematical models and algorithms help optimize robot performance and ensure reliable operation.

Adaptability and Autonomy: Mathematics enables robots to adapt to changing environments and perform tasks autonomously.

Others

Robotic Arms: Mathematical models are used to control the movements of robotic arms in manufacturing, assembly, and other industrial applications.

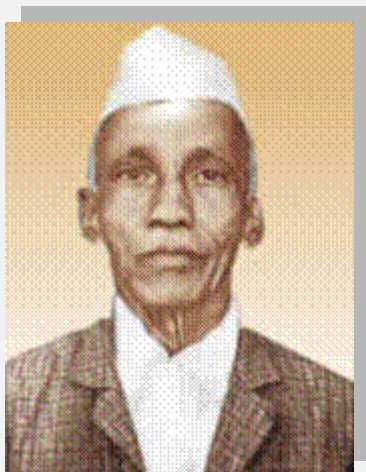
Autonomous Vehicles: Mathematics is essential for navigation, path planning, and obstacle avoidance in autonomous vehicles.

Medical Robotics: Mathematical principles are used to design and control robotic systems used in surgery and other medical procedures.

The seamless integration of mathematical techniques in robotics ensures that robots can operate efficiently, reliably, and safely in the real world.

ANCIENT INDIAN MATHEMATICIAN

R. D. KAPREKAR



ANCIENT INDIAN MATHEMATICIAN
R. D. KAPREKAR

Aakanksha Banjare

M.Sc. 2nd Sem.



Dattatreya Ramchandra Kaprekar (17 January 1905 – 1986) was an Indian recreational mathematician who described several classes of natural numbers including the Kaprekar, Harshad and self numbers and discovered Kaprekar's constant, named after him. Despite having no formal postgraduate training and working as a school teacher, he published extensively and became well known in the recreational Mathematics circle.

Kaprekar received his secondary school in Thane and studied at Fergusson College in Pune. In 1927, he won the Wrangler R. P. Paranjpye Mathematical Prize for an original piece of work in mathematics.

CONTRIBUTIONS

Kapreker's Constant:

In 1955, Kaprekar discovered an interesting property of the number 6174, which was subsequently named the Kaprekar constant. He showed that 6174 is reached in the end as one repeatedly subtracts the highest and lowest numbers that can be constructed from a set of four digits that are not all identical. Thus, starting with 1234, we have

$$4321 - 1234 = 3087, \text{ then}$$

$$8730 - 0378 = 8352, \text{ and}$$

$$8532 - 2358 = 6174.$$

Repeating from this point onward leaves the same number (7641 – 1467).

Kaprekar Number:

Another class of numbers Kaprekar described are Kaprekar numbers. A

Kaprekar number is a positive integer with the property that if it is squared, then its representation can be partitioned into two integer parts whose sum is equal to the original number (e.g. 45 since $45^2=2025$, and $20+25=45$, also 9, 55, 99 etc.) However,

note the restriction that the two numbers are positive; for example, 100 is not a Kaprekar number even though $100^2=10000$, and $100+00 = 100$. This operation, of taking the rightmost digits of a square, and adding it to the integer formed by the leftmost digits, is known as the Kaprekar operation.

Some examples of Kaprekar numbers in base 10, besides the numbers 9, 99, 999, ..., are

umber	Square	Decomposition
703	$703^2 = 494209$	$494+209 = 703$
2728	$2728^2 = 7441984$	$744+1984 = 2728$

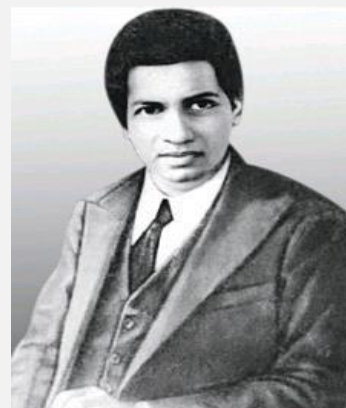
THE INDIAN MATHEMATICS: "SRINIVASA RAMANUJAN JI"

Geeta Tondan
M.Sc. 2nd Sem.



INTRODUCTION:-

Srinivasa Ramanujan was born on **22 December, 1887**, in Erode, India. A self-taught mathematician, He made significant contributions to number theory and mathematical analysis, despite facing limited formal education. He was born in a poor family. His father **Mr. Kuppaswamy Srinivasa Iyengar** worked as a clerk in a saree shop. His mother **Mrs. Komalatamma** was a house wife.



**Born : 22 December ,
1887, Erode, India**

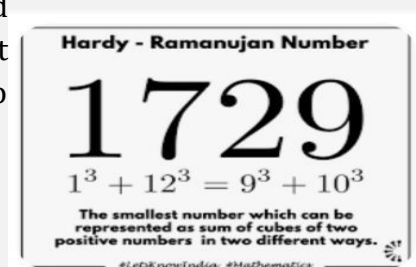
**Died: 2 April, 1920,
Kumbakonam (aged 32)**

EDUCATION:-

- **Early Life and Education :-** Ramanujan was born in Kumbakona Tamil - Nadu, India and attended local schools.
- **Self- Study :-** He developed his mathematical abilities through persistent self-study and research, even without formal mathematical training.
- **Mathematical Genius:-** His true mathematical genius emerged independently through his own efforts.
- **University of Madras :-** In 1903, he briefly attended the University Madras.

CONTRIBUTION'S:-

- **Infinite Series and Continued Fractions:-** Developed advanced formulas for hypergeometric series and discovered relationships between different series. Contributed to the theory of q-series and modular forms.
- **Ramanujan-Hardy Number(1729):-** Identified the famous number 1729 as the smallest positive integer expressible as the sum of two cubes in two different ways:
 $1729 = 13^3 + 12^3 = 9^3 + 10^3$



$$=13+123=93+103.$$

This incident led to the term "Taxicab Number."

- **Mock Theta Function:-** Introduced and studied mock theta functions, extending the theory of theta functions in modular forms.
- **Partition Function and Congruences:-** Investigated the partition function, yielding ground breaking results and congruences that significantly advanced number theory.
- **Ramanujan Prime and Tau Function:-** Proposed the concept of the Ramanujan prime, contributing to the understanding of prime numbers. Worked on theta function, providing insights into modular forms and elliptic functions.
- **Theta Functions and Elliptic Functions:-** Made profound contributions to the theory of theta functions and elliptic functions, impacting the field of complex analysis.
- **Unified Theories:-** Strived to unify different branches of mathematics, demonstrating a deep understanding of mathematical structures.
- **Collaboration with G.H.Hardy:-** collaborated with G. H. Hardy at Cambridge University, resulting in joint publications that enriched the field of mathematics.
- **Theorems in Calculus :-** Developed theorems in calculus, showcasing his ability to provide rigorous mathematical proofs for his intuitive results.

SASTRA RAMANUJAN PRIZE:-

This annual prize, awarded by the Shanmugha Arts, Science, Technology & Research Academy (**SASTRA**), honors individuals younger than **32 years** for their exceptional working mathematics.

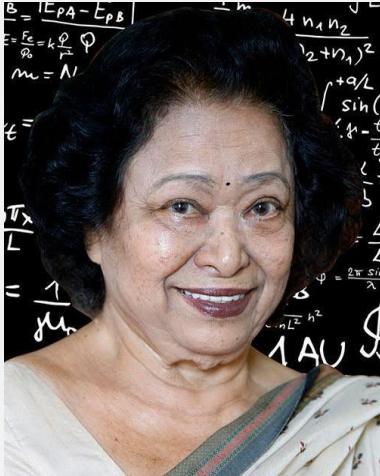
NATIONAL MATHEMATICS DAY:-

Every year, Ramanujan's birth anniversary on **22 December** is observed as "**National Mathematics Day**".

SRINIVASA RAMANUJAN'S MOST FAMOUS QUOTE IS:-

"An equation for me has no meaning unless it expresses a thought of God".

Ancient Indian Mathematician: Shakuntala Devi Ji



Anjika Tekam
M.Sc. 2nd Sem.

INTRODUCTION

Shakuntala Devi, The Human computer was born orthodox hindu family on 4 November 1929 (Monday), in the semislum. area in Gavipuram, Guttahalli, Bangalore, India.

As she was born in. a financially poor family, she had to struggle a lot. At the age of 3 when kids used to play with toys she used to play with playing cards and numbers .

At the age of 10, while she was studying, in school, she was been asked to leave as couldn't afford it, because of which she couldn't receive her formal education.

Her father Bishaw Mitra Mani circus performed was a road show who used to perform street shows in playing cards.

EDUCATION

Shakuntala. Devi, known as the " Human Computer ", did not receive any education, but her extraordinary Mathematical abilities Where evident from a young age, leading her to demonstrat her skills at the University of Mysore and travel the World showcasing her tallent.

HERE A MORE DETAILED LOOK AT HER EDUCATION:

Lack of formal education

Shakuntala devi never Attended school or received any formal education due to her family's financial constraints.

Early Talent:

Her exceptional mathematical abilities were recognized early on, with her father, who worked in a circus, noticing her talent for card tricks and calculations.

Global Recognition:

She traveled to various countries, including London, to demonstrate her mathematical skills and become known as the "Human Computer".

Self-Taught :

Despite her lack of formal education, she became a renowned mathematical and author, showcasing her remarkable skills and knowledge.

She authored books on mathematical and puzzle, inspiring countless minds.

Shakuntala Devi Books on math :

Puzzle To Puzzle You Books :

Puzzles to puzzle by Shakuntala Devi had made mathematics fun for those who fear from maths, thought, this book.

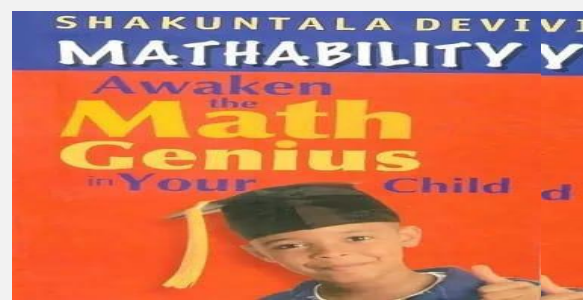
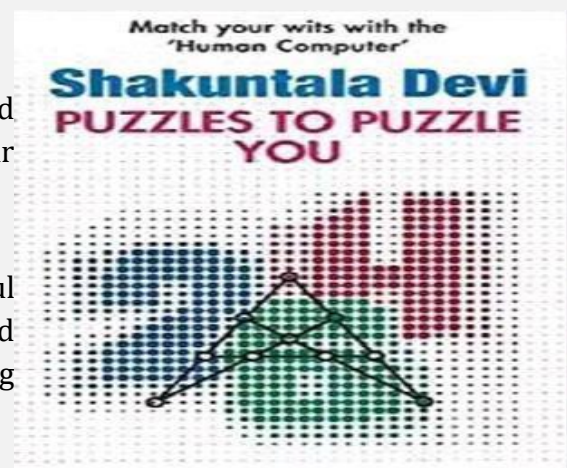
This puzzle book shows its reader delightful ways in which they can tease their brains and make them work thought solving some existing maths problems.

The Book Of Numbers

The book of numbers by Shakuntala Devi takes their readers to a new world of numbers. They will learn the things that had never known it. Teaches the shortcuts of maths, a lot of tips and tricks will be mentioned there, which one can quickly learn and use in their daily life. The book of number mainly focuses on eradicating the fear of maths from the minds of people.

Mathematical Merry – Go – Round

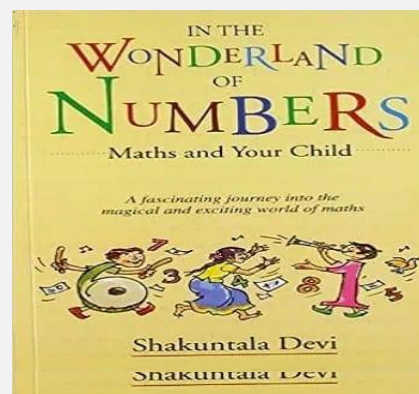
Mathematical merry go round Shakuntala Devi dramatizes the untending charm of



numbers and their ability to astonish and inspire. Fractions, decimals and compound interest become clear and easy. In this book she will teach easy to divide shortcuts on how to add long columns in your head, find square root, multiply, magically and quickly.

In The Wonderland Of Numbers

This book motivates the kids to do little mathematics tricks from 0 to 9. It motivates. Parents and teaches them to mold their kids to take the drive into the world of maths and to fall in love with it.



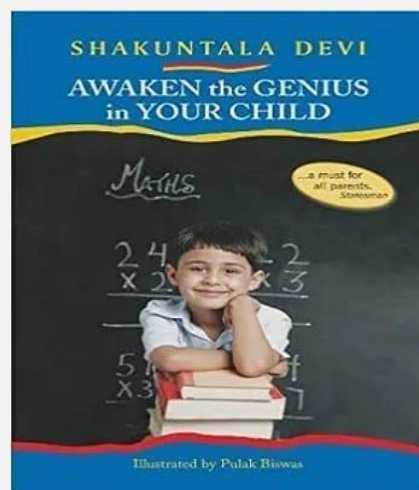
This book is the story of Neha, who is lagging in maths. Her new teacher fills her mind with the fear of maths, because of which she fails in the maths exams.

After the exam, on the way back to her home she meets. With an accident because of which she goes into a coma. She enters into the world of maths, also known as the kingdom of zero in her subconscious mind.

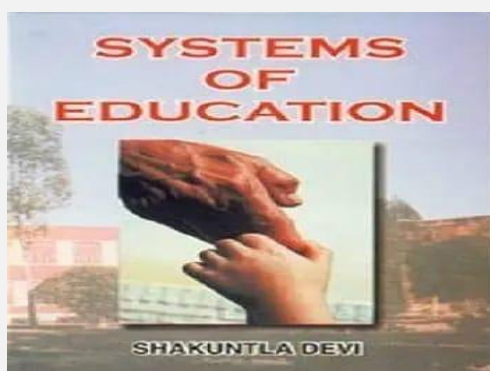
Mathability Awaken

The Math Genius In Your Child

in today's increasingly complex and technological world, the essential thing you can do for your kid is to nourish math ability. Fear of maths is weakening the child's future. Mathability is a skill that teaches a child how to think. Those who say 'that their child is poor at Maths' are doing themselves an injustice. Mathability is a skill that develops intelligence potential.



Systems To Education



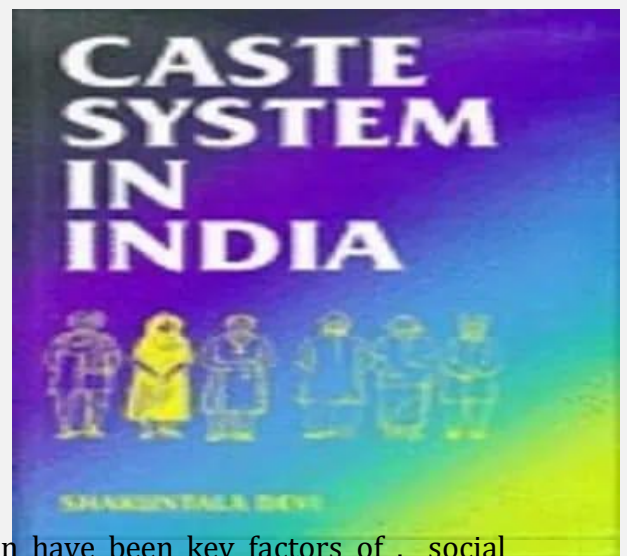
On the educational system in America and its implications for India. In this book, she compares the American education system. She highlights all the points that were missing in India that are implemented in America and vice versa.

She tells the importance of better education in india and how it can be improved. Thought she was being thrown out of school at the age of 10 as she couldn't pay her 2 rs fees and didn't even have her primary education, she had been various parts of the world and compared the education system, and she mentions the ways to improve it.

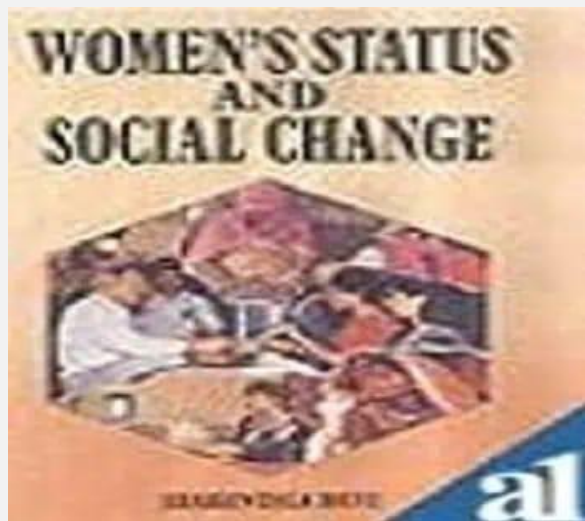
Cast System In India

In this book Shakuntala Devi presents everything that she had analyzed and researched. Few of the topics that are included are:

The changing concept of caste in India history and review India's social customs and systems. Society class, family and individual Division of castes
Disintegration and multiplication of caste.



Women's Status And Social Change



Women have been key factors of social changes as well, and because of social change, there is a huge change in the lives of people. Women stood arm to shoulder with men to change the society. In this book, Shakuntala Devi shares the importance of women's society and the changes they tried to bring in society how they bring changes in the lives of people.

Black Scholes model

The **Black-Scholes** or **Black-Scholes-Merton model** is a mathematical model for the dynamics of a financial market containing derivative investment instruments. From the parabolic partial differential equation in the model, known as the Black-Scholes equation, one can deduce the **Black-Scholes formula**, which gives a theoretical estimate of the price of European-style options and shows that the option has a *unique* price given the risk of the security and its expected return (instead replacing the security's expected return with the risk-neutral rate). The equation and model are named after economists Fischer Black and Myron Scholes. Robert C. Merton.

The main principle behind the model is to hedge the option by buying and selling the underlying asset in a specific way to eliminate risk. This type of hedging is called "continuously revised delta hedging" and is the basis of more complicated hedging strategies such as those used by investment banks and hedge funds.

The model is widely used, although often with some adjustments, by options market participants. The model's assumptions have been relaxed and generalized in many directions, leading to a plethora of models that are currently used in derivative pricing and risk management. The insights of the model, as exemplified by the Black-Scholes formula, are frequently used by market participants, as distinguished



Affara Khan
M.Sc. 4th Sem

from the actual prices. These insights include no-arbitrage bounds and risk-neutral pricing (thanks to continuous revision). Further, the Black-Scholes equation, a partial differential equation that governs the price of the option, enables pricing using numerical methods when an explicit formula is not possible.

The Black-Scholes formula has only one parameter that cannot be directly observed in the market: the average future volatility of the underlying asset, though it can be found from the price of other options. Since the option value (whether put or call) is increasing in this parameter, it can be inverted to produce a "volatility surface" that is then used to calibrate other models, e.g. for OTC (over the counter) derivatives.

The formula led to a boom in options trading and provided mathematical legitimacy to the activities of the Chicago Board Options Exchange and other options markets around the world. Merton and Scholes received the 1997 Nobel Memorial Prize in Economic Sciences for their work, the committee citing their discovery of the risk neutral dynamic revision as a breakthrough that separates the option from the risk of the underlying security. Although ineligible for the prize because

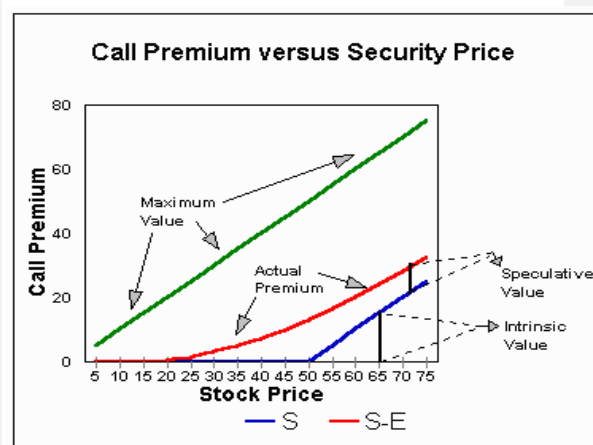
of his death in 1995, Black was mentioned as a contributor by the Swedish Academy.

The Black–Scholes model assumes that the market consists of at least one risky asset, usually called the stock, and one riskless asset, usually called the money market cash, or bond. The following assumptions are made about the assets (which relate to the names of the assets):

FUNDAMENTAL HYPOTHESES

- Risk-free rate: The rate of return on the riskless asset is constant and thus called the risk-free interest rate.
- Random walk: The instantaneous log return of the stock price is an infinitesimal random walk with drift; more precisely, the stock price follows a geometric brownian motion, and it is assumed that the drift and volatility of the motion are constant. If drift and volatility are time-varying, a suitably modified Black–Scholes formula can be deduced, as long as the volatility is not random.
- The stock does not pay a dividend.
- The assumptions about the market are:
- No arbitrage opportunity (i.e., there is no way to make a riskless profit).
- Ability to borrow and lend any amount, even fractional, of cash at the riskless rate.
- Ability to buy and sell any amount, even fractional, of the stock (this includes short selling).
- The above transactions do not incur any fees or costs (i.e., frictionless market).

- With these assumptions, suppose there is a derivative security also trading in this market. It is specified that this security will have a certain payoff at a specified date in the future, depending on the values taken by the stock up to that date. Even though the path the stock price will take in the future is unknown, the derivative's price can be determined at the current time. For the special case of a European call or put option, Black and Scholes showed that "it is possible to create a hedged position consisting of a long position in the stock and a short position in the option, whose value will not depend on the price of the stock". Their dynamic hedging strategy led to a partial differential equation which governs the price of the option. Its solution is given by the Black–Scholes formula.
- Several of these assumptions of the original model have been removed in subsequent extensions of the model. Modern versions account for dynamic interest rates (Merton, 1976) transaction cost and taxes (Ingersoll, 1976) and dividend payout.



Notation

The notation used in the analysis of the Black-Scholes model is defined as follows (definitions grouped by subject):

General and market related:

t is a time in years; with $t=0$ generally representing the present year.

r is the annualized risk-free interest rate continuously compounded (also known as the *force of interest*).

Asset related:

$s(t)$ is the price of the underlying asset at time t , also denoted as S_t .

μ is the drift rate of s annualized.

σ is the standard deviation of the stock's returns. This is the square root of the quadratic variation of the stock's log price process, a measure of its volatility

Option related:

$V(s,t)$ is the price of the option as a function of the underlying asset S at time t , in particular:

$C(s,t)$ is the price of a European call option and

$P(s,t)$ is the price of a European put option.

T is the time of option expiration.

τ is the time until maturity: $\tau=T-t$.

K is the strike price of the option, also known as the exercise price.

$N(x)$ denotes the standard normal cumulative distribution function.

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{z^2}{2}} dz$$

$N'(x)$ denotes the standard normal probability density function

$$N'(x) = \frac{dN(x)}{dx} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Black-Scholes Equation

The Black-Scholes equation is a parabolic partial differential equation that describes the price $V(s,t)$ of the option, where S is the price of the underlying asset and t is time:

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

A key financial insight behind the equation is that one can perfectly hedge the option by buying and selling the underlying asset and the bank account asset (cash) in such a way as to "eliminate risk". This implies that there is a unique price for the option given by the Black-Scholes formula.

Mathematician Of The Day (11th January)

Fischer Black was an American mathematical economist, best known as one of the authors of the famous Black-Scholes equation. In 1997, the Nobel Prize for Economics was awarded jointly to Myron Scholes (Fischer Black's co-author of the paper on option pricing) and to Robert C. Merton (another pioneer in the development of the valuation of stock-options). A Nobel Prize is not awarded posthumously but Fischer Black would undoubtedly have been a joint winner of the 1997 Nobel Prize for Economics had he lived. In their announcement of the 1997 Prize, the Nobel Committee paid tribute to Black's key role. Black was born on this day (January 11) in 1938.

By Mamun-Or-Rashid

Assistant Professor (Mathematics), BGCTUB



Fischer Black
(January 11, 1938
– August 30, 1995)

Myron S. Scholes

Fischer Black

Robert C. Merton

